Ceres [20 pts]

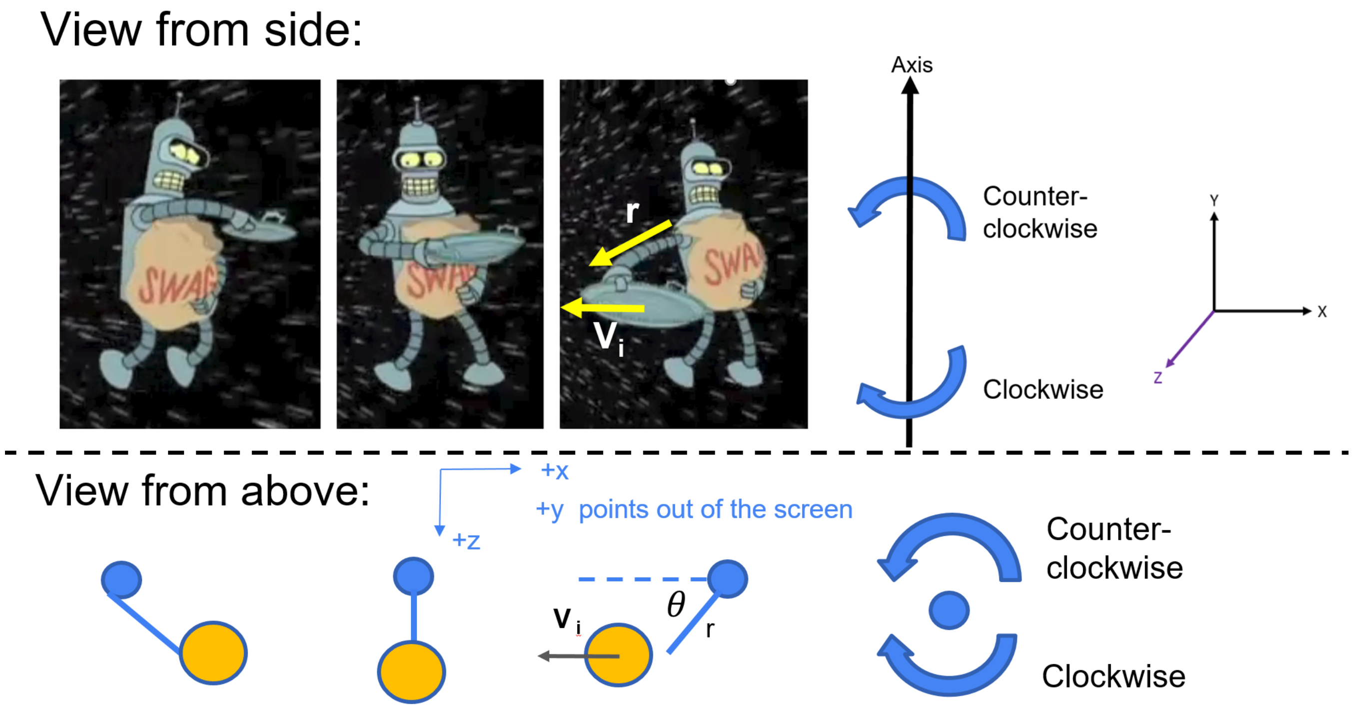
The minor planet Ceres has a mass of $M = 9 \times 10^{20}$ kg and a radius of 473 km. The Tycho Manufacturing company is working to create sustainable colonies in the Asteroid Belt, and Ceres is among their first locations. The first steps in the work involve sending raw materials from Ceres to the Anderson Space Station, located one million kilometers away, for processing into usable goods. In this problem, you can ignore the Sun and all the other objects in the Solar System.

1. [10 pts] Tycho engineers need to launch a 200 kg package of raw materials from Ceres to Anderson Station. How fast does the package need to be launched, if it needs to have a speed of 3 m/s when it arrives at Anderson?

2. [10 pts] To send the package, Tycho engineers built a **horizontal** spring-loaded launching mechanism on Ceres. When the apparatus is ready to engage, the package is at rest and the spring is compressed a distance of 1.5 m. What is the stiffness of the spring?

Bender [20 pts]

Bender is in space, far away from anything, and carrying a bag of swag. He tosses a plate of mass $m = 2$ kg with his right arm, and when the plate leaves his hand there's an angle $\theta = 30$ degrees as seen in the figure.



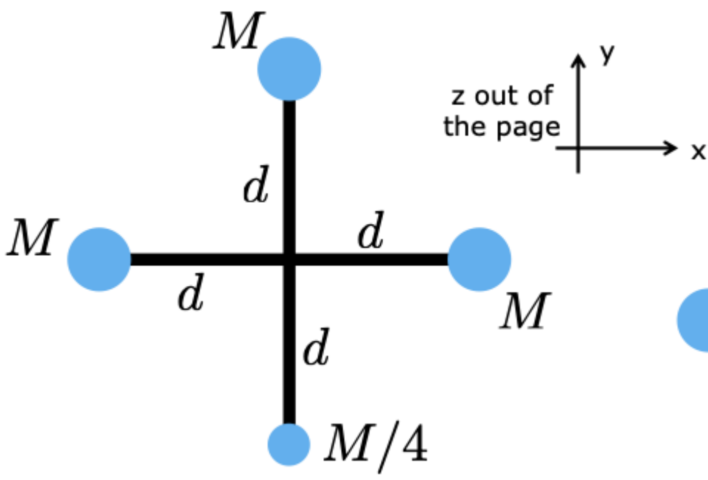
1. [5 pts] Which direction will Bender rotate after tossing the plate, clockwise or counterclockwise? Explain using the angular momentum principle and the right hand rule.

2. [10 pts] If the speed of the plate is 5 m/s when Bender tosses it, and Bender's fully-extended arm is one meter long, what would be Bender's final angular momentum? Note that Bender's moment of inertia is 150 kg m^2, and that there's an angle involved (see bottom part of the figure).

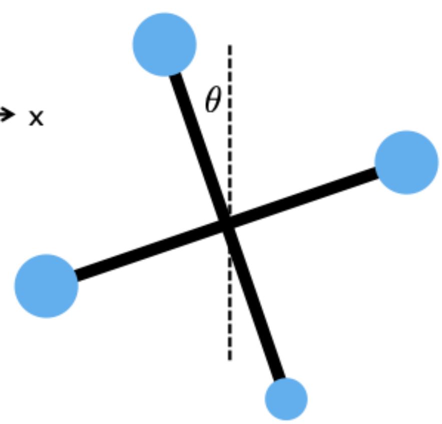
3. [5 pts] Bender's getting dizzy and wants to stop spinning now. The remaining swag in the bag has a mass of 4 kg. How fast and in which direction should he toss the remaining swag so that he stops rotating altogether?

Rods [20 pts]

A system is composed of four identical massless rods, each with length d and four balls attached to the ends of each rod. Three of the balls have mass M and the fourth one has mass $M/4$. The system is free to rotate about its center, where the bars come together. **You can think of the balls as point masses.** Earth's gravity points in the usual direction ($-\hat{y}$).



(Figure 1)



(Figure 2)

1. [5 pts] What is the net torque on the system as shown in Figure 1, where the rods are oriented along the x and y axes?

2. [5 pts] You hold the system at rest after rotating it by a small angle θ counterclockwise (as shown in Figure 2). Then you let go. What will happen to the system now? Use the angular momentum principle to justify your answer.

3. [10 pts] Determine the final angular momentum of the system T seconds after letting it go from the initial configuration in Figure 2.

EXTRA CREDIT [5 pts]

1. [1 pt] Write the equation for the Momentum Principle. To earn credit your equation must be fully correct.

2. [1 pt] Write the equation for the Energy Principle. To earn credit your equation must be fully correct.

3. [1 pt] Write the equation for the Angular Momentum Principle. To earn credit your question must be fully correct.

4. [2 pts] The Lyman Alpha line appears in the emission line spectrum of hydrogen when the electron goes from the $N = 2$ to the $N = 1$ level, emitting a photon in the process. What is the energy of the Lyman Alpha photon?

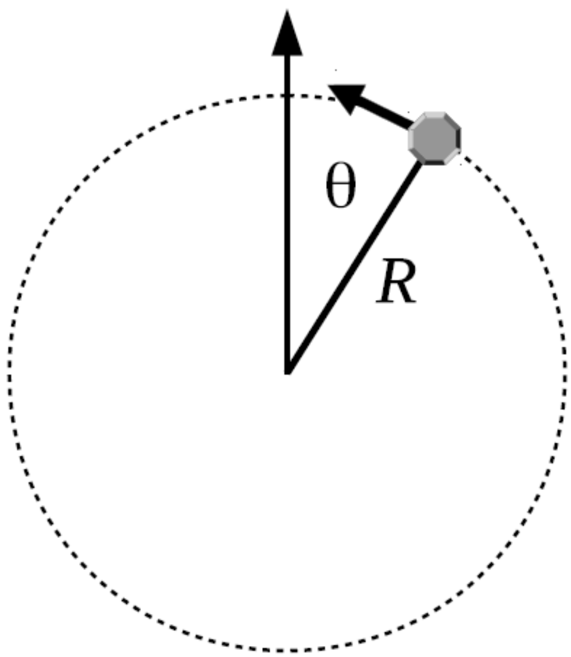
   - 0 eV
   - 3.4 eV
   - 6.8 eV
   - 10.2 eV
   - 13.6 eV

PHYS 2211 Modern Final Review Worksheet – Reading Day Fall 2021

Curving motion / Gravitation

Problem 1

On the Earth, a rock of mass m is tied to the end of a rope of length R. The rock is swung counterclockwise in a circle of radius R in a vertical plane (gravity points down). Consider the rock when the rope makes an angle of θ with the vertical, as shown in the diagram. At this location, the tension in the rope is a known quantity T and the speed of the rock is decreasing.



(a 5pts) Is the perpendicular component of the net change in momentum for the rock zero? Explain briefly how you know this.

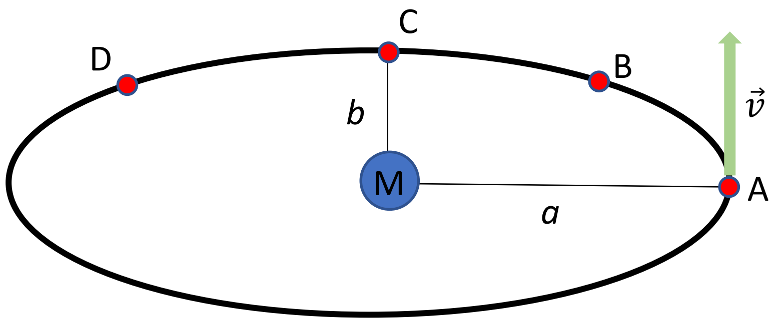
(b 5pts) Is the parallel component of the net change in momentum for the rock zero? Explain briefly how you know this.

(c 7pts) Determine the parallel component of the net force on the rock in terms of the known quantities given in the problem statement.

(d 8pts) Calculate the speed of the rock in terms of the known quantities given in the problem statement.

Problem 2

A planet orbits a star in an elliptical orbit, shown below. The ellipse has major axis of $2a$ and a minor axis of $2b$. The smallest radius of curvature occurs at the point on the orbit furthest from the center, when it equals $\frac{b^2}{a}$. The largest radius of curvature occurs at the point on the orbit closest to the center, when it equals $\frac{a^2}{b}$. Assume the gravity from the star is the only force acting on the planet.

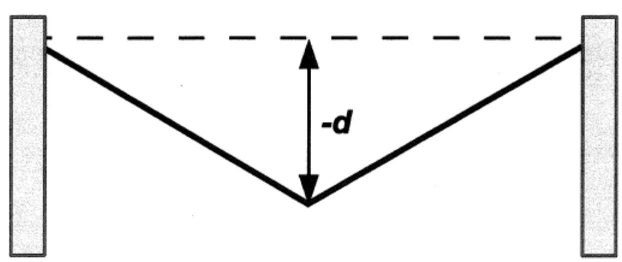


a) At which point(s) will $\frac{d\overrightarrow{p_{||}}}{dt}$ of the planet be zero?
b) At which point(s) will $\frac{d\overrightarrow{p_{\perp}}}{dt}$ of the planet be zero?
c) At which point(s) will $\frac{d\overrightarrow{p_{||}}}{dt}$ of the planet be negative?
d) At which point(s) will $\frac{d\overrightarrow{p_{\perp}}}{dt}$ of the planet be negative?
e) At which point(s) will the planet have the greatest speed?
f) *Bonus* Find the ratio the greatest and lowest speeds of the object during its orbit.

Energy principle

Problem 3

A trampoline is composed of a strong fabric sheet connected to a frame. When the trampoline is stretched at the center a distance $y = -d$, as shown in the figure, it pushes back with a force $\vec{F}_{tramp} = +kd^3\hat{y}$ and has a potential energy $U = \frac{1}{4}kd^4$. Here k is a positive constant unique to the trampoline.

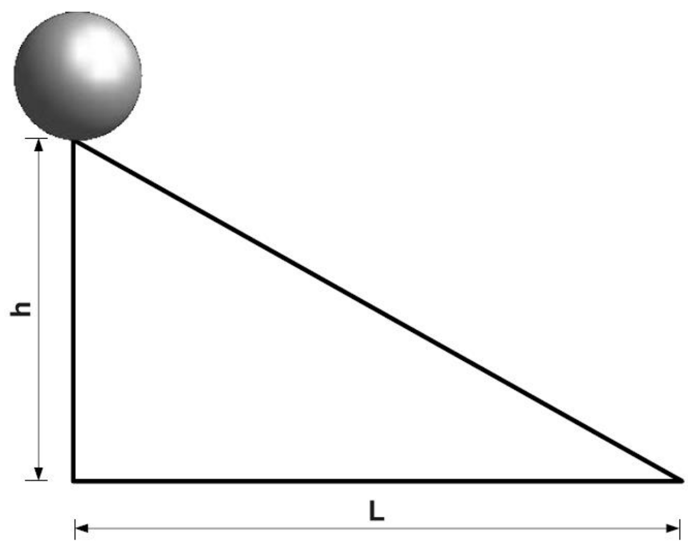


(a 10pts) A bowling ball of mass m is gently placed onto the un-stretched trampoline. After waiting a few seconds the bowling ball comes to rest and the trampoline is stretched an amount d_{static}. Taking the bowling ball as your system, how much work did the trampoline do as the ball traveled from $d = 0$ to $d = d_{static}$?

(b 15pts) In a different experiment, a bowling ball of mass m is dropped from rest a height h above the un-stretched trampoline (d=0). Determine the maximum stretch d_{max} of the trampoline during the bounce of the bowling ball.

Problem 4

A solid sphere of mass M and radius R rolls without slipping down a hill as shown in the figure. The lack of slipping is the result of a friction force which causes the sphere to rotate at a rate $\omega = v_{CM}/R$, where v_{CM} is the velocity of the center of mass.



(a 5pts) Choose your system to be the sphere only. As the sphere rolls down the hill, determine the work done on the real system.

(b 10pts) The sphere starts at the top of the hill from rest and rolls to the bottom. Determine the speed of the sphere when it reaches the bottom. Apply the energy principle on the real system consisting only of the sphere.

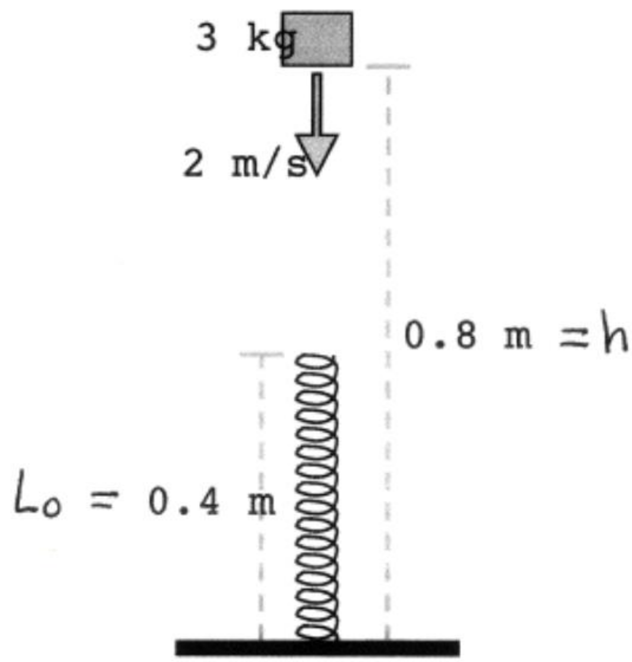
(c 5pts) Determine the average frictional force acting on the sphere as it rolled to the bottom of the hill. Apply the energy principle on the point particle system consisting only of the sphere. You should use your result from part (a).

(d 5pts) A second sphere with the same mass total M and radius R but with a hollow core is rolled down the hill. This hollow sphere will have a higher density then the solid sphere so that they have the same mass. Does it have a final speed (at the bottom) that is greater than, equal to or less than the solid sphere from part (a)? Briefly explain how you know this.

Energy graphs

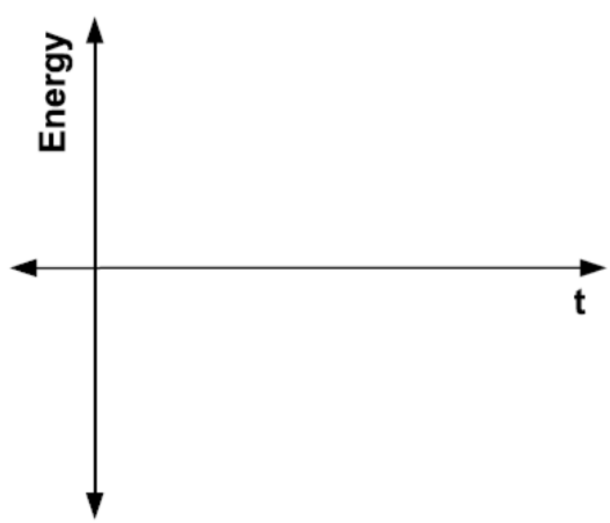
Problem 5

A metal block of mass 3 kg is moving downward with speed 2 m/s when the bottom of the block is 0.8 m above the floor. When the bottom of the block is 0.4 m above the floor, it strikes the top of a relaxed vertical spring 0.4 m in length. The stiffness of the spring is 2000 N/m.



a) The block continues downward. When the bottom of the block is 0.3 m above the floor, what is its speed? (Consider the block, the spring, and earth as the system).

(c 5pts) Sketch a graph of spring potential energy, gravitational potential energy, and the kinetic energy as a function of time.

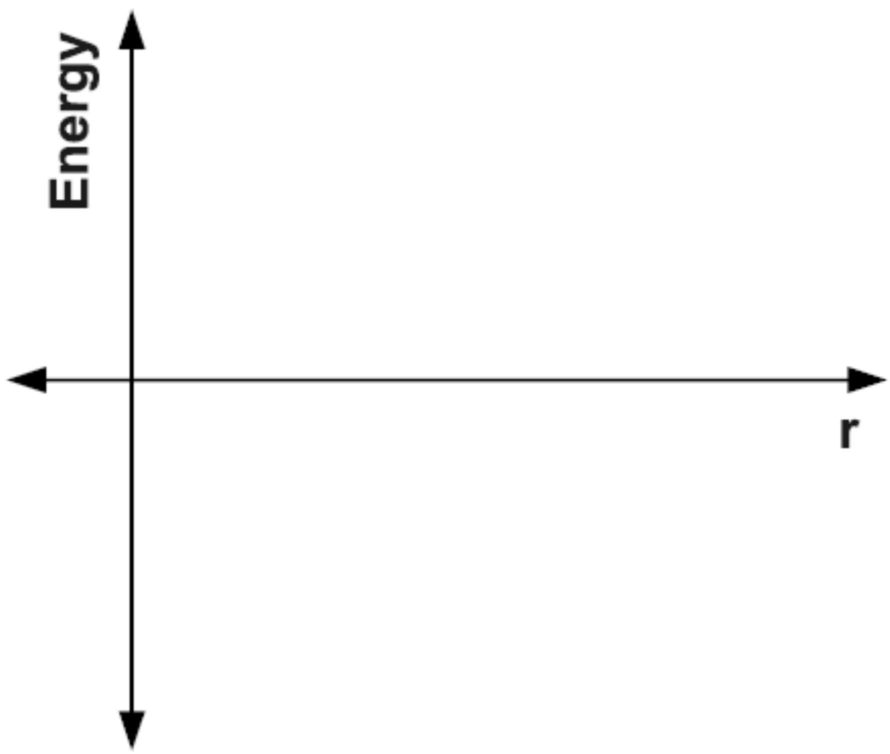


Problem 6

A rock of mass m is released from rest very far from the Earth (i.e. $r = \infty$). The only force acting on the rock is the gravitational force of the Earth. In the following questions you can assume the mass of the rock is much less than the mass of the Earth and neglect air resistance.

(a 5pts) Determine the velocity of the rock the instant it reaches the surface of the Earth. Your answer should not be numeric.

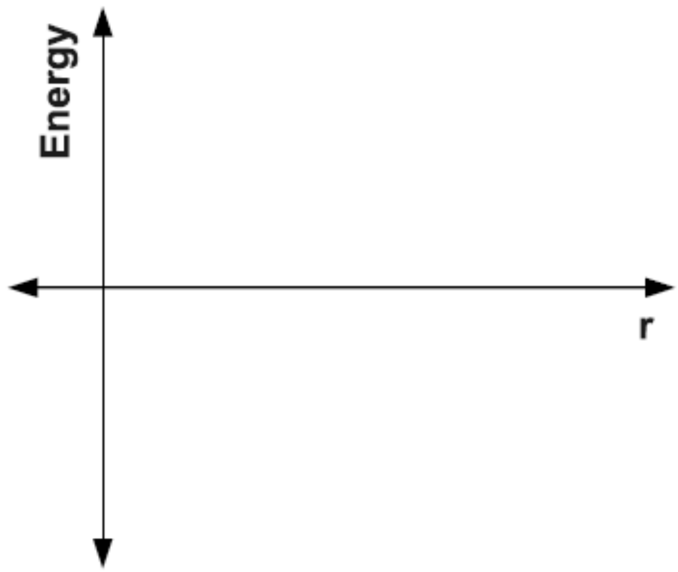
(b 5pts) For the case considered above, sketch the: Kinetic, Potential, and Total energy for the Earth+Rock system.



A rock with mass m is released from rest at the Earth's surface and drops through a hole all the way to the center of the Earth. The magnitude of the gravitational force on the rock is mgr/R, where r is the rock's distance from the center of the Earth and R is the Earth's radius. This force points towards the center of the Earth and is the only force acting on the rock. In the following questions you can assume the mass of the rock is much less than the mass of the Earth and neglect air resistance.

(c 10pts) Determine the velocity of the rock the instant it reaches the center of the Earth. Your answer should not be numeric.

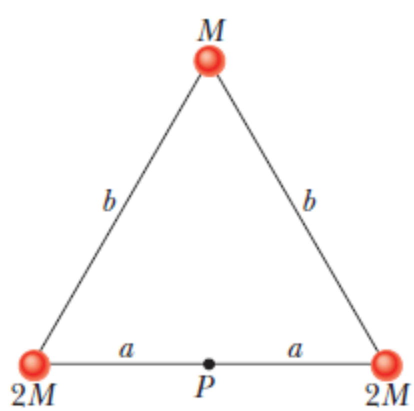
(d 5pts) For the case of the rock falling through the Earth, the potential energy of the Earth+Rock system is given by $mgr^2/(2R)$. On the graph below, sketch the Kinetic, Potential and Total energy for the Earth+Rock system.



(Extra credit 5pts) How does your answer to part (c) compare to your answer in part (a)? Hint: the ratio of the two velocities should not depend on any of the parameters in the problem.

Angular momentum and Torque for solid extended system

Problem 7

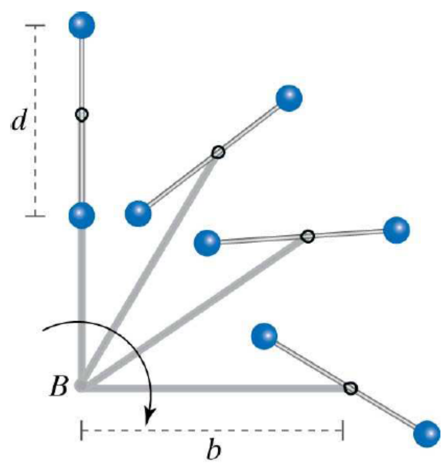


The rigid body shown in the figure consists of three particles connected by massless rods. It is to be rotated about an axis perpendicular to its plane through point P.

a) What is the moment of inertia of the body about the axis through point P?
b) A constant torque is applied and the body which is rotated about P. After a time Δt, it has an angular velocity of ω. What is the magnitude of the applied torque?

Problem 8

A barbell consists of two small balls, each with mass m, at the ends of a very low mass rod of length d. The center of mass for the barbell is mounted on the end of a low mass rigid rod of length b. As shown in the diagram, the rod rotates clockwise with angular speed ω. In addition, the barbell rotates counterclockwise about its own center, with an unknown angular speed.



(a 10pts) Determine the translational angular momentum $\vec{L}_{trans,B}$ (magnitude and direction) for the barbell.

(b 5pts) If the total angular momentum for this system (about the point B) is zero, calculate the rotational angular momentum for the barbell about its center of mass. $\vec{L}_{rot,cm}$ (magnitude and direction)

(c 5pts) Calculate the moment of inertia I for the barbell about its center of mass.

(d 5pts) Determine the unknown angular speed of the barbell about its center of mass.

Angular Momentum and Torque for a system with particles

Problem 9

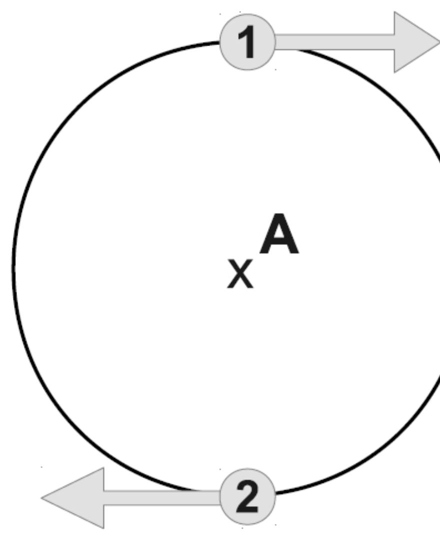
Consider a system consisting of three particles:

$m_1 = 05$ kg, $\vec{r}_1 = \langle 5,\ 0,\ 0\rangle$ m, $\vec{v}_1 = \langle -4,\ -12,\ 0\rangle$ m/s
$m_2 = 20$ kg, $\vec{r}_2 = \langle 0,\ 5,\ 0\rangle$ m, $\vec{v}_2 = \langle 0,\ 0,\ -5\rangle$ m/s
$m_3 = 10$ kg, $\vec{r}_3 = \langle 0,\ 0,\ 5\rangle$ m, $\vec{v}_3 = \langle 8,\ 0,\ -16\rangle$ m/s

(a 5pts) Determine the angular momentum of particle 1 relative to the point $\vec{A} =< 0, 0, 0 >$ m.?

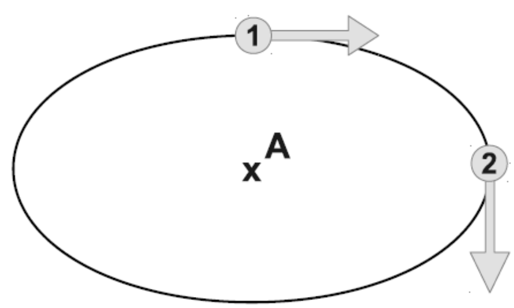
(b 5pts) Determine the angular momentum of particle 3 relative to the point $\vec{A} =< 10, 0, 5 >$ m.?

(c 5pts) A race car drives clockwise around a circular track at a constant speed as seen in the figure. Determine the direction of the angular momentum of the car relative to the center of the track.



(d 5pts) How does the angular moment of the car, relative to the center of the track, change as the car goes from location 1 to location 2? Your answer should be supported by physics principles.

(e 5pts) A race car drives clockwise around an elliptical track at a constant speed as seen in the figure. How does the angular moment of the car, relative to the center of the track, change as the car goes from location 1 to location 2? Your answer should be supported by physics principles.



Problem 10

Consider a system consisting of three particles:

$m_1 = 5$ kg, $\vec{r}_1 = \langle 5,\ 0,\ 0 \rangle$ m, $\vec{v}_1 = \langle 5,\ -10,\ 15 \rangle$ m/s
$m_2 = 10$ kg, $\vec{r}_2 = \langle 0,\ 5,\ 0 \rangle$ m, $\vec{v}_2 = \langle -10,\ 0,\ -5 \rangle$ m/s
$m_3 = 15$ kg, $\vec{r}_3 = \langle 0,\ 0,\ 5 \rangle$ m, $\vec{v}_3 = \langle 0,\ 0,\ -10 \rangle$ m/s

A) What is the angular momentum of particle 2 relative to the origin?

B) What is the angular momentum of particle 3 relative to the location of particle 2?

C) BONUS: What is the total angular momentum of the system about the origin? *(Just set up the equations unless you want to do a long calculation! It is good practice for doing cross products, if that is something you want.)*

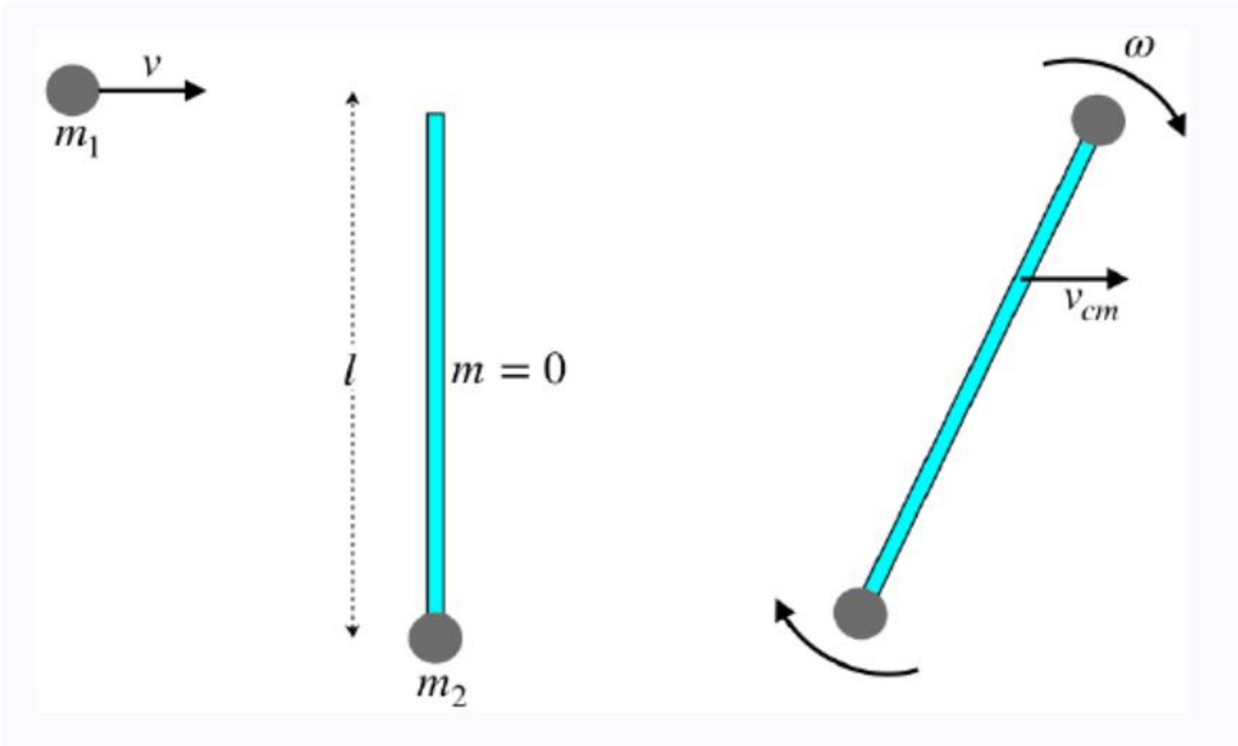
Additional Practice

Problem 11 – Moment of Inertia / Conceptual

In which of the following cases does Object 2 have a greater moment of inertia than Object 1 about their given axes? Select all that apply

Choice	Object 1	Object 2
A	A sphere of mass M about an axis through its center of mass	A hollow sphere of mass M about an axis through its center of mass
B	A sphere of density δ about an axis through its center of mass	A hollow sphere of density δ about an axis through its center of mass
C	A solid cube of density δ and side length *s* about an axis through an edge.	A solid cube of density δ and side length 2*s* about an axis through an edge.
D	A cone of mass M about an axis perpendicular to the cone's base and passing through its tip.	A cone of mass M about an axis parallel to cone's base and passing through its tip.
E	Three balls each of mass M connected by massless rods of length l about an axis through the center of one of the balls.	Three balls each of mass M connected by massless rods of length l about an axis through the center of the formed triangle.

Problem 12 – Conservation of Angular Momentum



A massless magnetic rod has a small steel ball (which does have mass, but a negligible radius) attached to one end, and is at rest. Another small steel ball approaches the open end of this rod at a right-angle, and when it reaches the end of the rod, sticks to it. The dumbbell-looking combination continues forward, spinning as it goes (see the diagram). **Prove this collision elastic.** The diagram provides labeling of quantities that you can use – you cannot make any assumptions about the relative values of m_1 and m_2.

Problem 13 – Energy Conservation

A positively charged sphere moves through an electric field produced by an oppositely charged particle at location r_{source} = <1, 1, 0> m. Initially, the sphere is at rest at position r_1 = <0,0,0>. After some time, it reaches location r_2 =< .25, .5, 0> m with a translational momentum of <120, 120,0> kg*m/s and an unknown spin, ω. The magnitude of charge on the spheres is 2 mC. The moving sphere's mass is 1 kg and it has radius of .2 meter.

Reminder: The moment of inertia of a sphere is $\frac{2}{5}MR^2$. The potential energy of an electric particle is $k\frac{q_1q_2}{r}$.

What is the magnitude of the angular velocity, ω, of the sphere at location r_2?