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Math 1554 Linear Algebra Spring 2024

Midterm 3 (C) Make-up

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

Name: ________________________________ GTID Number: ____________________

Student GT Email Address: ________________________________@gatech.edu

Section Number (e.g. A3, G2, etc.) ________ TA Name __________________________

Circle your instructor:

Prof Barone Prof Belegradek Prof Kumar Prof Sun

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 9 pages of questions.

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You do not need to justify your reasoning for questions on this page.

1. (a) (8 points) Suppose A is a real $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and $\vec{b}$. Otherwise, select **false**.

true	false	
○	○	If A is an $n \times n$ real diagonalizable matrix, then A has n distinct eigenvalues.
○	○	If $\text{proj}_{\vec{u}}(\vec{v}) = \vec{0}$, then $\{\vec{u}, \vec{v}\}$ is linearly independent.
○	○	If $A = PDP^{-1}$ is diagonalizable and invertible, then A^{-1} is diagonalizable with the same matrix D.
○	○	If A has QR-factorization $A = QR$, then $\text{Nul}(A^T) = \text{Nul}(Q^T)$.
○	○	If $A\vec{x} = \vec{x}$ and $A\vec{y} = 2\vec{y}$ then the matrix $B = [\vec{x}\ \vec{y}]$ has rank 2.
○	○	Every 4×4 real matrix A has a real eigenvalue.
○	○	If U is an $n \times n$ orthogonal matrix, then $UU^T = I_n$.
○	○	If A is $n \times n$ and A has n distinct real eigenvectors, then A is diagonalizable.

You do not need to justify your reasoning for questions on this page.

(b) (4 points) Indicate whether the following situations are possible or impossible.

possible	impossible	
◯	◯	A 2×2 matrix A such that A is an orthogonal matrix and $\det(A) = -1$.
◯	◯	A 3×3 matrix in RREF that has exactly one pivot and is not diagonalizable.
◯	◯	A real 5×5 matrix A whose eigenvalues are precisely $i, 1, 1 + i, 1 - i, 0$.
◯	◯	A real 5×5 matrix A with exactly two real eigenvalues, and the two eigenspaces of A have dimension 2 and dimension 3, respectively.

(c) (2 points) Which of the following are examples of a matrix A which satisfy the following property. The matrix A is not diagonalizable while the matrix A^2 is diagonalizable. *Select all that apply.*

◯ $\begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}$

◯ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

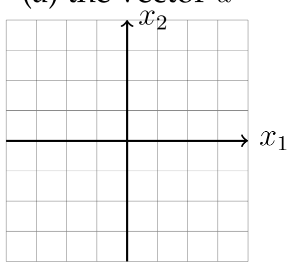
◯ $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

◯ $\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

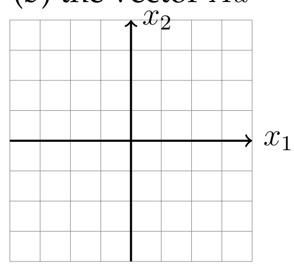
You do not need to justify your reasoning for questions on this page.

2. (5 points) Consider $A = \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$, $\vec{u} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. Sketch (a) the vector $\vec{u}$ and (b) the vector $A\vec{u}$. Then, fill in the blanks for the following statements to make the statements true.

(a) the vector $\vec{u}$



(b) the vector $A\vec{u}$



(c) The transformation defined by $A = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$ is a rotation-dilation where the rotation is through $\boxed{}$ (degrees or radians counter-clockwise) and the scaling factor is $\boxed{}$.

(d) An eigenvector associated to the eigenvalue $\lambda = -1+i$ of A is the vector $\vec{x} = \begin{pmatrix} \boxed{} \\ 1 \end{pmatrix}$.

3. (3 points) Fill in the blanks so that the following statements are true. Let $A = PDP^{-1}$ where

$$P = \begin{pmatrix} 2 & 2 & 3 \\ 1 & 1 & 3 \\ 2 & 1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 3 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

(a) An eigenvector of A corresponding to eigenvalue $\lambda = 4$ is $\vec{x} = \begin{pmatrix} \boxed{} \\ 3 \\ \boxed{} \end{pmatrix}$.

(b) The vector $\vec{x} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix}$ is an eigenvector of A corresponding to eigenvalue $\lambda = \boxed{}$.

(c) The characteristic polynomial of A is equal to $\boxed{}$.

You may leave your answer in factored form.

You do not need to justify your reasoning for questions on this page.

4. (6 points) Fill in the blanks.

   (a) Suppose $\vec{u}$ and $\vec{v}$ are vectors in $\mathbb{R}^n$ and that $\vec{u}\cdot\vec{v} = -4$, $\|\vec{u}\| = \sqrt{6}$ and $\|\vec{v}\| = 2\sqrt{2}$. Determine the length of $\vec{2}u - 3\vec{v}$.

$$\|2\vec{u} - 3\vec{v}\| = \boxed{}$$

   (b) A 3×2 matrix such that $\mathrm{Col}(A)^{\perp}$ is spanned by $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

$$A = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$$

   (c) Write $\vec{y} = \vec{x} + \vec{z}$ as the sum of two vectors, where $\vec{x}$ in span$\{\vec{u}\}$ and $\vec{z}$ is orthogonal to $\vec{u}$.

$$\vec{y} = \begin{pmatrix} 5 \\ 3 \end{pmatrix} \qquad \vec{u} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \qquad \vec{x} = \begin{pmatrix} \\ \end{pmatrix} \qquad \vec{z} = \begin{pmatrix} \\ \end{pmatrix}$$

You do not need to justify your reasoning for questions on this page.

5. (4 points) Let W be the subspace in $\mathbb{R}^4$ which is the set of solutions to the following homogeneous linear equation. Find $\dim W$ and an orthonormal basis for $W^{\perp}$.

$$x_1 - x_2 + x_3 - 3x_4 = 0$$

$\dim W = \boxed{}$

orthonormal basis for $W^{\perp}$ is $\boxed{}$

You do not need to justify your reasoning for questions on this page.

6. (4 points) The matrix A has the QR factorization $A = QR$. Find the least-squares solution of the linear system $A\vec{x} = \vec{b}$ when

$$Q = \frac{1}{13}\begin{pmatrix} 5 & 12 \\ 0 & 0 \\ -12 & 5 \end{pmatrix}, \qquad R = \begin{pmatrix} 1 & -2 \\ 0 & 3 \end{pmatrix}, \qquad \vec{b} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix}.$$

$\hat{x}$= $\boxed{}$

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7. (4 points) Set $A = \begin{pmatrix} 0 & 0 \\ 2 & 3 \\ 4 & 6 \end{pmatrix}$. Find a basis for each of the four fundamental subspaces $\mathrm{Col}(A)$, $\mathrm{Nul}(A)$, $\mathrm{Row}(A)$ and $\mathrm{Nul}(A^T)$.

basis for $\mathrm{Col}(A)$ is

basis for $\mathrm{Nul}(A)$ is

basis for $\mathrm{Row}(A)$ is

basis for $\mathrm{Nul}(A^T)$ is

You do not need to justify your reasoning for questions on this page.

8. (5 points) **Show all work for problems on this page.**

   Find the vector $\vec{x}$ which is in W and closest to $\vec{y}$, where the subspace W is spanned by $\vec{v}_1$ and $\vec{v}_2$. Then, compute the distance from $\vec{y}$ to W.

$$\vec{y} = \begin{pmatrix} 3 \\ 3 \\ 0 \\ 3 \end{pmatrix}, \qquad \vec{v}_1 = \begin{pmatrix} 1 \\ 3 \\ -1 \\ 1 \end{pmatrix}, \qquad \vec{v}_2 = \begin{pmatrix} 3 \\ -1 \\ 1 \\ 1 \end{pmatrix}.$$

$\vec{x}$= $\boxed{}$

$\text{dist}(\vec{y}, W) =$ $\boxed{}$

9. (5 points) **Show all work for problems on this page.**

   Compute the parameters α and β of the least-squares curve $y = \alpha x + \beta \cos(\frac{\pi x}{3})$ that best fits the data below.

x	-2	0	1
y	-1	0	2

$\alpha = \boxed{}$ $\beta = \boxed{}$

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