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Math 1554 Linear Algebra Spring 2024

Midterm 1 (C) Make-up

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

Name: _____ GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A3, G2, etc.) _____ TA Name _____

Circle your instructor:

Prof Barone Prof Belegradek Prof Kumar Prof Sun

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- **Simplify your answers** unless explicitly stated otherwise.
- **Fill in circles** completely. Do not use check marks, X's, or any other marks.
- Calculators, notes, cell phones, books are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.
- The last page is for scratch work. Please use it if you need extra space.
- This exam has 7 pages of questions.

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You do not need to justify your reasoning for questions on this page.

1. (a) (8 points) Suppose A is an $m \times n$ matrix and $\vec{b} \in \mathbb{R}^m$ unless otherwise stated. Select **true** if the statement is true for all choices of A and $\vec{b}$. Otherwise, select **false**.

true	false	
◯	◯	If the last row of the RREF of a matrix A is all zeros, then $A\vec{x} = \vec{0}$ has infinitely many solutions.
◯	◯	If $\vec{x} \neq \vec{0}$ and $T(\vec{x}) = \vec{0}$, then T is not one-to-one.
◯	◯	If the linear transformation $T(\vec{x}) = A\vec{x}$ is one-to-one, then $A\vec{x} = \vec{b}$ has a unique solution.
◯	◯	If A is a 2×2 matrix that represents a clockwise rotation by 180 degrees and I is the 2×2 identity matrix, then $A^2 = I$.
◯	◯	If the linear system $A\vec{x} = \vec{b}$ is inconsistent, then its augmented matrix $[A \mid \vec{b}]$ has a pivot in each column in its echelon form.
◯	◯	The set of solutions to the linear system below is a line in $\mathbb{R}^4$. $\begin{cases} x_1 - 2x_2 + x_3 + x_4 = 0 \\ -x_1 + 3x_2 - x_3 + x_4 = 0 \end{cases}$
◯	◯	If $\vec{v}$ is a solution to an inhomogeneous system $A\vec{x} = \vec{b}$, then the vector $\vec{v} + \vec{v}$ is also a solution to $A\vec{x} = \vec{b}$.
◯	◯	If A, B are $n \times n$ matrices, then $B(A + I)B = B^2A + B^2$.

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(b) (4 points) Indicate whether the following situations are possible or impossible.

possible	impossible	
◯	◯	The transformation $T : \mathbb{R}^n \to \mathbb{R}^m$ defined by $T(\vec{x}) = A\vec{x}$ is one-to-one, but T is not onto.
◯	◯	A transformation $T : \mathbb{R}^2 \to \mathbb{R}^3$ that is one-to-one.
◯	◯	A homogeneous linear system which has solutions that are not the trivial solution.
◯	◯	Vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ such that $\{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$ is linearly dependent and none of the vectors $\vec{v}_1, \vec{v}_2, \vec{v}_3$ are scalar multiples.

(c) (2 points) Let T and S be linear transformations of $\mathbb{R}^2$ such that T is the projection onto the line $x_2 = 0$ and S is the projection onto the line $x_1 = 0$. Which of following accurately describes the transformation $S(T(\vec{x}))$? Select only one.

- ◯ Counterclockwise rotation by 180 degrees about the origin.
- ◯ Projection onto the line $x_1 = 0$.
- ◯ Projection onto the line $x_2 = 0$.
- ◯ The transformation that sends every vector to zero.

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(d) (4 points) For the vectors

$$\vec{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, \ \vec{v}_2 = \begin{pmatrix} 2 \\ -2 \\ 2 \end{pmatrix}, \ \vec{v}_3 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}.$$

Which of the following sets are linearly independent? *Select all that apply.*

- ◯ $\{\vec{v}_1, \vec{v}_2\}$
- ◯ $\{\vec{v}_2, \vec{v}_3\}$
- ◯ $\{\vec{v}_1, \vec{v}_3\}$
- ◯ $\{\vec{v}_1, \vec{v}_1 + \vec{v}_2\}$

2. (3 points) If possible, fill in the box with the missing element of the vector $\vec{x}$ so that $\vec{x}$ is in the set span$\{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$. If it is not possible write `NP` in the space.

$$\vec{u}_1 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, \ \vec{u}_2 = \begin{pmatrix} 3 \\ -1 \\ -2 \end{pmatrix}, \vec{u}_3 = \begin{pmatrix} -1 \\ 5 \\ 0 \end{pmatrix}, \vec{x} = \begin{pmatrix} -2 \\ -4 \\ \boxed{} \end{pmatrix}$$

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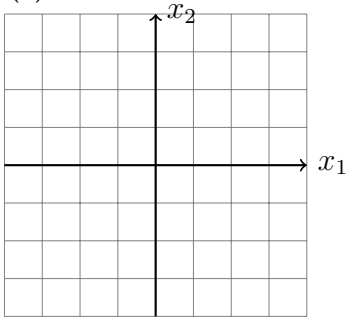
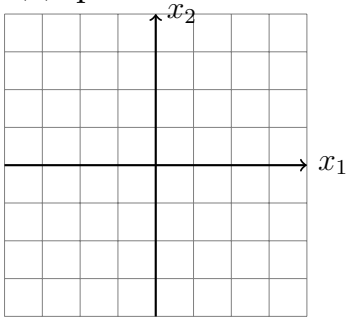
3. (4 points) Suppose $A = \begin{pmatrix} -4 & 1 \\ 8 & -2 \end{pmatrix}$ and sketch (a) a non-zero solution with integer entries to $A\vec{x} = \vec{0}$, and (b) the span of the columns of A.

(a) non-zero solution



(b) span of columns



4. (4 points) Consider the linear system in variables x_1, x_2, x_3 with unknown constants below.

$$a_1x_1 + a_2x_2 + a_3x_3 = b_1$$
$$a_1x_1 + a_2x_2 + a_3x_3 = b_2$$

Which of the following statements about the solution set of this system are possible? *Select all that apply.*

- ◯ The solution set is empty.
- ◯ The solution set is a single point.
- ◯ The solution set is a line.
- ◯ The solution set is a plane.

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5. (8 points) **Show your work for part (c) and put your answer in the boxes.**
Let T be the linear transformation defined by

$$T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 - x_2 - 3x_3 \\ x_1 - 2x_3 \\ x_1 + x_2 - x_3 \end{bmatrix}.$$

(a) What are the domain and codomain of T?

domain is ☐

codomain is ☐

(b) Find the matrix A such that $T(\vec{x}) = A\vec{x}$ for all $\vec{x}$.

$A =$ ☐

(c) Find a vector $\vec{x}$ such that $T(\vec{x}) = \vec{b}$, where $\vec{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

$\vec{x} =$ ☐

(d) Is T one-to-one? ◯ yes ◯ no

(e) Is T onto? ◯ yes ◯ no

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6. (7 points) **Show all work for problems on this page.**

   For what value(s) of h is the following set of vectors linearly dependent?

$$\left\{ \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} h \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ -3 \\ h^2 \end{pmatrix} \right\}$$

$$h = \boxed{}$$

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7. **Show your work in the space below the first box and put your answers in the boxes.**

(a) (6 points) Write the parametric vector form for the general solution to the inhomogeneous equation $A\vec{x} = \vec{b}$ given below.

$$\begin{cases} 5x_1 + x_2 - 7x_3 - 9x_4 = 2 \\ 5x_1 + x_2 - 4x_3 - 6x_4 = -1 \end{cases}$$

(b) For the homogeneous system with the same coefficient matrix A as part (a) above, write down the parametric vector form of the general solution to $A\vec{x} = \vec{0}$.
Hint: use your answer from part (a).

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