Math 1552, Integral Calculus

Review for Test #3

Sections 8.8, 10.1-10.5

****NOTE FROM THE INSTRUCTORS: PRIOR METHODS OF INTEGRATION THAT WE HAVE STUDIED, WHILE NOT SPECIFICALLY INDICATED ON THIS REVEW SHEET, MAY BE REQUIRED TO SOLVE IMPROPER INTEGRALS.****

1. Content Recap

(a) An integral $\int_a^b f(x)dx$ is *improper* if at least one of the limits of integration is _____________, or if there is a _____________ _____________ on the interval $[a, b]$.

(b) A sequence **converges** if _____________ and **diverges** if _____________.

(c) A geometric series has the general form _____________.
The series converges to _____________ when _____________ and diverges when _____________.

(d) A p-series has the general form _____________. The series converges when _____________ and diverges when _____________. To show these results, we can use the _____________ test.

(e) The harmonic series has the general form _____________, and it always _____________!

(f) To find the sum of a telescoping series, we should first break it into ___________ ___________.

(g) The series $\sum a_n$ diverges if the limit is NOT equal to _____________.

(h) If $\lim_{n\to\infty} a_n = 0$, then what, if anything, do we know about the series $\sum_n a_n$?
____

(i) To apply the integral test, we must have a function that is _____________, _____________, and _____________. We then know that the integral $\int_a^\infty f(x)\ dx$ and the series $\sum_{n=a}^\infty f(n)$ both _____________ or both _____________.

(j) If you want to show a series converges, compare it to a ____________ series that also converges. If you want to show a series diverges, compare it to a ____________ series that also diverges.

(k) If the direct comparison test does not have the correct inequality, you can instead use the ____________ ____________ test. In this test, if the limit is a ____________ number (not equal to ____________), then both series converge or both series diverge.

(l) In the ratio and root tests, the series will ____________ if the limit is less than 1 and ____________ if the limit is greater than 1. If the limit equals 1, then the test is ____________.

2. Determine if each integral below converges or diverges, and evaluate the convergent integrals.

(a) $\int_1^\infty \frac{\ln(x)}{x^2}\,dx$

(b) $\int_7^\infty \frac{dx}{x^2-x}$

(c) $\int_6^\infty \frac{3t^2}{\sqrt{t^3-8}}\,dt$

(d) $\int_2^6 \frac{3t^2}{\sqrt{t^3-8}}\,dt$

(e) $\int_2^\infty \frac{3t^2}{\sqrt{t^3-8}}\,dt$

(f) $\int_0^3 \frac{x}{(x^2-1)^{2/3}}dx$

(g) $\int_1^\infty \frac{1-\ln(x)}{x^2}dx$

3. Determine if each infinite series converges or diverges. If it converges, **find the sum**.

(a) $\sum_{n=0}^\infty e^{-3n}$

(b) $\sum_{n=1}^\infty \frac{4^n+5^n}{9^{n-1}}$

(c) $\sum_{n=0}^\infty \ln\left(\frac{n+5}{n+6}\right)$ (find an expression for S_n, the n^{th} partial sum of this series, and then take the limit to determine if it converges)

(d) $\sum_{n=0}^\infty \cos(5\pi n)$ (find an expression for S_n, the n^{th} partial sum of this series, and then take the limit to determine if it converges)

(e) $\sum_{n=1}^\infty \left(\frac{1}{\sqrt{n+3}} - \frac{1}{\sqrt{n+5}}\right)$ (find an expression for S_n, the n^{th} partial sum of this series, and then take the limit to determine if it converges)

(f) $\sum_{n=3}^\infty \frac{3^{n-2}+2^{n+1}}{5^{n+1}}$

(g) $\sum_{n=4}^{\infty} \frac{1}{(3n+1)(3n+4)}$ (find an expression for S_n, the n^{th} partial sum of this series, and then take the limit to determine if it converges)

4. Determine whether the sequences converge or diverge. Find the limit of each convergent sequence.

(a) $\left\{\left(1-\frac{1}{n^2}\right)^n\right\}$

(b) $\left\{(10n)^{\frac{1}{n}}\right\}$

(c) $\left\{\frac{2n+(-1)^n}{4+3n}\right\}$

5. Determine whether the given series converges or diverges. Make sure that you (1) name the test and state the conditions needed for the test you are using, (2) show work for the test that requires some math, and (3) state a conclusion that explains why the test shows convergence or divergence.

(a) $\sum_{n=2}^{\infty} \frac{n+1}{\sqrt{n^3-2}}$

(b) $\sum_{n=3}^{\infty} \frac{10}{n \ln n \ln(\ln n)}$

(c) $\sum_{n=1}^{\infty} \frac{n+2}{\sqrt{n^5+4}}$

(d) $\sum_{n=1}^{\infty} \frac{n+3}{\sqrt{n^2+1}}$

(e) $\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} + \ldots$

(f) $\sum_{n=1}^{\infty} ne^{-3n^2}$

(g) $\sum_{n=1}^{\infty} \frac{3(n!)^2}{(2n)!}$

(h) $\sum_{n=1}^{\infty} \frac{n}{(\ln n+7)^n}$

(i) $\sum_{n=1}^{\infty} \left(\frac{n+2}{n}\right)^{n^2}$

(j) $\sum_{n=1}^{\infty} \frac{e^n}{n^e}$

(k) $\sum_{n=1}^{\infty} \frac{(2^n)^n}{n!}$

ANSWERS

2. (a) 1; (b) $\ln\left(\frac{7}{6}\right)$; (c) diverges; (d) $8\sqrt{13}$; (e) diverges; (f) $\frac{9}{2}$; (g) 0

3. (a) $\frac{e^3}{e^3-1}$, (b) $\frac{369}{20}$, (c) diverges
(d) diverges, (e) $\frac{1}{2} + \frac{1}{\sqrt{5}}$, (f) $\frac{41}{750}$; (g) $\frac{1}{39}$

4. (a) 1, (b) 1, (c) $\frac{2}{3}$

5. (c), (e), (f), (g), (h) converge