Math 1552, Integral Calculus
Study Questions for Test #1
Sections 5.1-5.6

1. **Formula Recap**: complete each of the following formulas.
(a) The general Riemann Sum is found using the formula:

(b) Some helpful summation formulas are:

$$\sum_{i=1}^{n} c =$$

$$\sum_{i=1}^{n} i =$$

$$\sum_{i=1}^{n} i^2 =$$

(c) Properties of the definite integral:

$$\int_a^a f(x)dx =$$

$$\int_b^a f(x)dx =$$

$$\int_a^b cf(x)dx =$$

(d) State the Fundamental Theorem of Calculus:

(e) Using the FTC:

$$\frac{d}{dx}\left[\int_{a(x)}^{b(x)} f(t)dt\right] =$$

(f) If F is an antiderivative of f, that means:

(g) If F is an antiderivative of f, then:

$$\int f(g(x))g'(x)dx =$$

(h) To find the area between two curves, use the following steps:

2. Fill in the integration formulas below:

$$\int x^n dx, \quad (n \neq -1) =$$

$$\int \sin(ax)dx =$$

$$\int \cos(ax)dx =$$

$$\int \sec^2(ax)dx =$$

$$\int \sec(ax)\tan(ax)dx =$$

$$\int \csc(ax)\cot(ax)dx =$$

$$\int \csc^2(ax)dx =$$

$$\int \frac{1}{1+(ax)^2}dx =$$

$$\int \frac{1}{\sqrt{1-(ax)^2}}dx =$$

$$\int \frac{1}{x}dx =$$

$$\int e^{ax}dx =$$

$$\int b^{ax}dx =$$

$$\int \tan x dx =$$

$$\int \sec x dx =$$

$$\int \csc x dx =$$

$$\int \cot x dx =$$

3. True or false? Determine if each statement below is *always* true or *sometimes* false.

(a) If F and G are both anti-derivatives of the continuous function f, then $F(x) = G(x)$.

(b) Let $F(x) = \int_{a(x)}^{b} f(t)\, dt$. Then $F'(x) = -f(a(x)) \cdot a'(x)$.

(c) Suppose f is a continuous function on $[a, b]$ and $\int_a^b f(x)\, dx = 0$. Then $f(x) = 0$ for all $x \in [a, b]$.

(d) Suppose that f and g are both continuous on $[a, b]$, and that $f(x) > g(x)$. Then $\int_a^b |f(x)|\, dx > \int_a^b |g(x)|\, dx$.

(e) If f is a continuous, increasing function, then the right-hand Riemann sum method always overestimates the definite integral.

(f) Let f be a continuous function and $av(f)$ be the average of f. Then $av(f) \cdot (b - a) = \int_a^b f(x)dx$.

(g) If $\int_0^1 f(x)\, dx = 9$ and $f(x) \geq 0$, then $\int_0^1 \sqrt{f(x)}\, dx = 3$.

4. Evaluate the following integrals using any method we have learned.

(a) $\int \sec^2(t) e^{1+\tan(t)}\, dt$

(b) $\int \sin(4x) \cos^3(4x)\, dx$

(c) $\int \frac{1}{\sqrt{4-9w^2}}\, dw$

(d) $\int \frac{6}{\sqrt{y}(5+6\sqrt{y})^5} dy$

(e) $\int_{-3}^{3} \sin(x^5) dx$

(f) $\int \frac{\sqrt{\ln x}}{x}\, dx$

(g) $\int x^2 e^{x^3}\, dx$

5. Let $F(x) = \int_x^{x^{1/5}} \frac{e^t}{t^5}\, dt$. Then the value of $F'(1)$ is:
(A) $\frac{e^5}{5}$ (B) $-e$ (C) 0 (D) $-\frac{4e}{5}$

6. Find $F'(4)$ if

$$F(x) = \int_{\frac{x^2}{4}}^{x^2} \ln(\sqrt{t})\ dt.$$

7. The velocity of a particle is given by the formula $v(t) = 5t^2 + 3t - 6$, in meters per second.

(a) Evaluate the actual distance traveled between time $t = 1$ and $t = 3$ seconds by dividing the interval into n equal subintervals and taking a limit of Riemann Sums, where x_i^* is chosen to be the right-hand endpoint of each subinterval.

(b) Check your answer to part (a) by calculating the actual value of $\int_1^3 v(t)dt$ using the FTC.

8. Suppose $f(x)$ is continuous and $f(x) < 0$ for all $x \in [a, b]$. Which of the following statements is *always* true?
(A) $\int_b^a f(x)\ dx > 0$.
(B) $\int_a^b f(x)\ dx > 0$.
(C) $\int_a^b f(x)\ dx = f(b) - f(a)$.
(D) $\int_a^b f(x)\ dx = 0$.

9. Rewrite the Riemann sum below as a definite integral.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \left(\left(3 + \frac{i}{n}\right) \left(\left(3 + \frac{i}{n}\right)^2 - 10 \right)^4 \right)$$

10. Use the midpoint rule with 4 subintervals to approximate the average of the function $f(t) = \frac{1}{2} + \sin^2(\pi t)$ on the interval $[0, 2]$.

11. Give an upper bound and a lower bound for the area under the curve $y = x^2$ over the interval $[0, 2]$ by partitioning into four subintervals.

12. The expression $\lim_{n\to\infty} \left(\frac{2}{n} \sum_{i=1}^{n} \left|-1 + \frac{2}{n}i\right|\right)$ represents the definite integral of a function f on the interval $[-1, 1]$. What is the value of this integral?
(A) 0 (B) 1 (C) 2 (D) $\frac{1}{2}$

13. Suppse that f is an even function; i.e., $f(-x) = f(x)$. What is $\int_{-a}^{a} f'(x)\ dx$?
(A) $2f(a)$
(B) $f(2a)$
(C) 0
(D) not enough information to solve

14. Select the correct expression for the midpoint sum that approximates the area bounded by $f(x) = \sin(x)$ on the interval $[0, \frac{\pi}{2}]$ using $n = 5$ rectangles of equal width.

(A) $\sum_{i=1}^{5} \sin\left(\frac{\pi i}{2}\right) \cdot \frac{\pi}{2}$

(B) $\sum_{i=1}^{5} \sin\left((i-1)\frac{\pi}{2}\right) \cdot \frac{\pi}{10}$

(C) $\sum_{i=1}^{5} \sin\left(\frac{\pi i}{20} - \frac{\pi}{2}\right) \cdot \frac{\pi}{10}$

(D) $\sum_{i=1}^{5} \sin\left(\frac{\pi i}{10} - \frac{\pi}{20}\right) \cdot \frac{\pi}{10}$

15. Find the area bounded by the curves $y = -x^2 + 6x$ and $y = x^2 - 2x - 24$. (Hint: sketch the curves or make a sign chart.)

16. Find the area of the triangle with vertices at the points (0,1), (3,4), and (4,2). USE CALCULUS.

Answers

3. (b), (e), (f) are true

4. (a) $e^{1+\tan(t)} + C$

(b) $-\frac{1}{16}\cos^4(4x) + C$

(c) $\frac{1}{3}\arcsin\left(\frac{3w}{2}\right) + C$

(d) $-\frac{1}{2(5+6\sqrt{y})^4} + C$

(e) 0

(f) $\frac{2}{3}(\ln x)^{\frac{3}{2}} + C$

(g) $\frac{1}{3}e^{x^3} + C$

5. D

6. $14\ln 2$

7. $\frac{130}{3}$ square units

8. A

9. $\int_3^4 x(x^2-10)^4\,dx$

10. 1

11. Upper Bound=$\frac{15}{4}$, Lower Bound=$\frac{7}{4}$

12. B

13. C

14. D

15. $\frac{512}{3}$ or approximately 170.67

16. 4.5 square units