

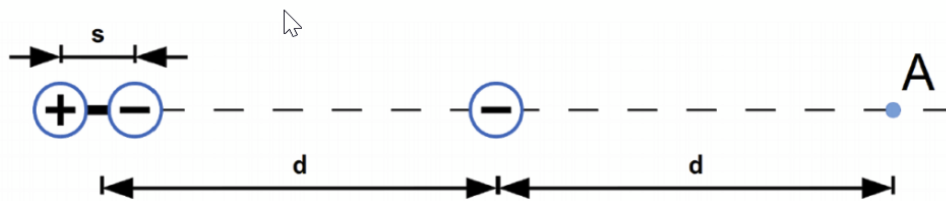
Physics 2212 Modern Studypalooza!!

PROBLEMS

ELECTRIC FIELDS AND FORCES

ELECTRIC FIELDS AND FORCES 1

A permanent dipole and a charged particle lie on the x-axis and are separated by a distance d as indicated in the figure. The dipole consists of positive and negative charge q separated by a distance s where $s \ll d$. The particle has a negative charge $-q$ and is located at $x = 0$.

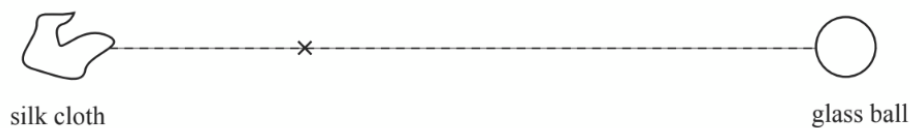


- Determine the magnitude and direction of the force on the point charge due to the permanent dipole. To earn full credit, you must show your work.
- What is the magnitude and direction of the net electric field at location A, a distance d to the right of the charged particle?
- A neutral atom that consists of positive and negative charge $2q$ is placed at location A. This neutral atom has a known polarization α . Sketch the polarized atom at location A and determine the separation length of the induced dipole.

ELECTRIC FIELDS AND FORCES

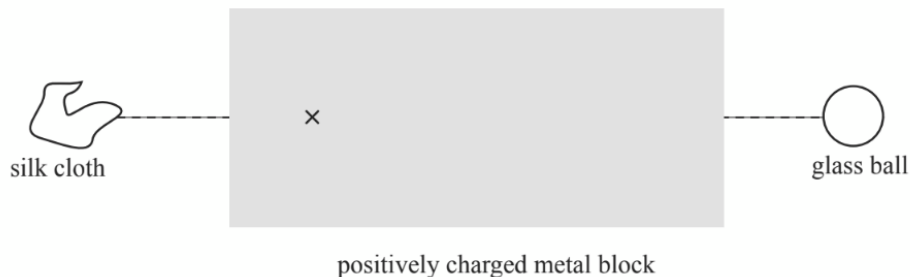
A small glass ball is rubbed all over with a small silk cloth and acquires a charge of $+5 \text{ nC}$. The silk cloth and the glass ball are placed 30 cm apart. *If any of the electric fields you are asked to draw is zero, state this explicitly.*

(a 4pts) At the location marked "x," draw and label two arrows, representing the electric field due to the silk cloth and the electric field due to the glass ball. The relative lengths of these arrows should be correct.



Now a *positively charged* metal block is placed between the two objects.

(b 5pts) Draw the approximate charge distribution for the positively charged metal block on the diagram below.

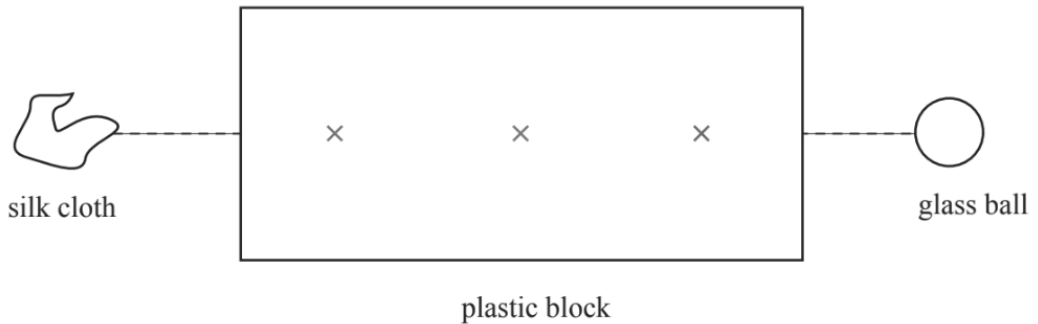


(c 10pts) At the location marked "x" draw *and label* four arrows, using the same scale as in part (a):

- The electric field due to the silk cloth \vec{E}_{cloth}
- The electric field due to the glass ball \vec{E}_{ball}
- The electric field due to the charges in and/or on the metal block \vec{E}_{metal}
- The net electric field \vec{E}_{net}

Now the metal block is removed, and replaced by a neutral plastic block.

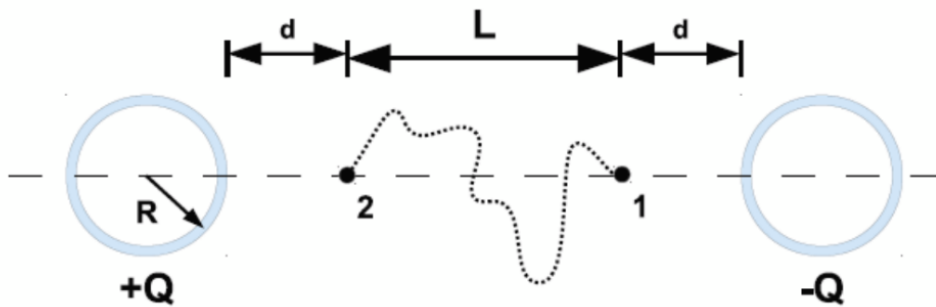
(g 6pts) Show the polarization of a molecule at each location marked "x" below.



ELECTRIC POTENTIAL

ELECTRIC POTENTIAL 1

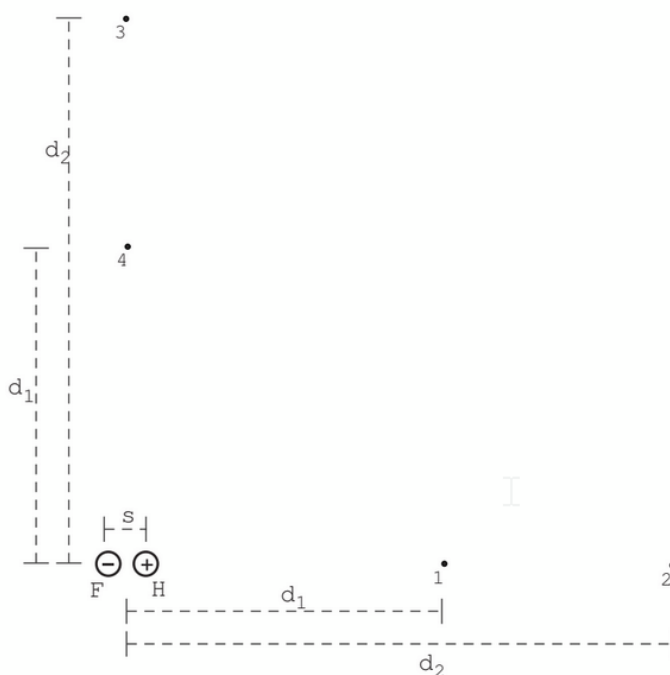
A thin spherical glass shell of radius R carries a uniformly distributed charge $+Q$. An identical shell carries a uniformly distributed charge $-Q$. The distance between the center of each shell is $2(R+d)+L$. Calculate the potential difference $V_1 - V_2$ between the two shells along the indicated meandering path (as shown in the diagram). Be sure to show all of your work to earn partial credit.



ELECTRIC POTENTIAL 2

An HF molecule is composed of two point charges, H^+ and F^- , with charges $+e$ and $-e$ respectively separated by a distance s (i.e. and electric dipole).

- (40 points) Calculate the potential difference $V_2 - V_1$, where locations 1 and 2 are shown on the diagram. The distances d_1 and d_2 are much larger than the internuclear separation s . Make sure the sign of your answer is correct. Show all of your work.



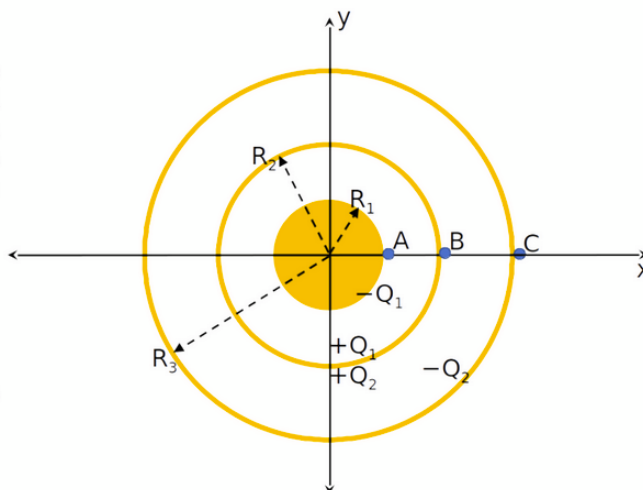
- (20 points) A proton is released from rest at location 1. Calculate the speed of the proton when it is very far (i.e. $r = \infty$) from the dipole. Show all of your work.
- (20 points) What is the potential difference $V_4 - V_3$? Locations 3 and 4 are on a line extending through the midpoint of the molecule. Show all of your work.
- (20 points) Calculate the potential difference $V_1 - V_4$, where locations 1 and 4 are shown on the diagram.

ELECTRIC POTENTIAL 3

As shown in the figure, a solid metal sphere of radius R_1 and charge $-Q_1$ is surrounded by two concentric spherical metal shells with radii R_2 and R_3 , respectively. The charges on these concentric spherical metal shells are:

- $+Q_1$ on the inner surface of R_2 ,
- $+Q_2$ on the outer surface of R_2 ,
- $-Q_2$ on the inner surface of R_3 ,
- no charge on the outer surface of R_3 .

All charges are distributed uniformly across inner and outer surfaces.



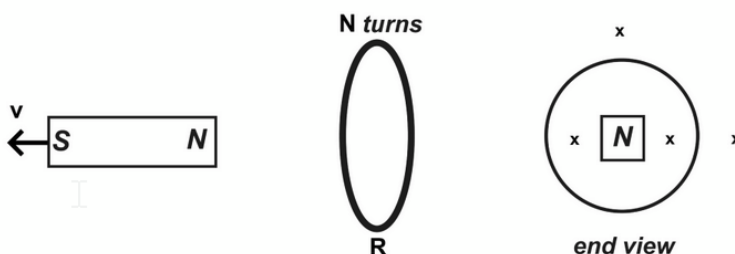
- [3 pts] At point C just outside the spherical shell R_3 , what is the electric potential V_C (that is, calculate $V_C - V_\infty$)? Note that when calculating the potential difference near infinitesimally thin shells, it does not matter if one of the end points is on or just outside the surface of the shell.

- B. [10 pts] Point A is just outside the metal sphere of radius R_1 . What is the potential difference, $V_A - V_C$? Show your work.
- C. [8 pts] What is the electric potential V_O at the center of the metal sphere (that is, calculate $V_O - V_\infty$)? Hint: You should use your results from parts A and B and think about how the charge would be distributed over the metal sphere.
- D. [4 pts] Suppose that $V_A = V_C$, as would be the case when the two components are connected by a metal wire. In this case, what is the ratio Q_1/Q_2 ?

FARADAY'S LAW

FARADAY'S LAW 1

In one experiment, the north pole of a bar magnet points toward a thin circular coil of wire containing N turns of radius R . The magnet is then moved away from the coil along the axis of the coil as shown in the diagram. In the following questions you should assume that, at any given time, the magnetic field produced by the magnet is uniform over the area enclosed by the loop.

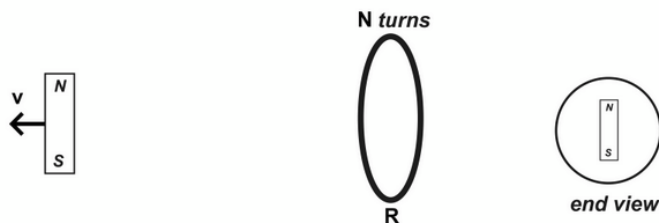


- A. [20 pts] Draw and label the following arrows (relative lengths of the arrows should scale):

- \vec{B}_0 the magnetic field at the center of the coil before the magnet was moving
- \vec{B}_t the magnetic field at the center of the coil while the magnet is moving
- $\frac{d\vec{B}}{dt}$ the change in the magnetic field as the magnet is moving
- indicate the direction of \vec{E}_{nc} at the "x" locations inside the coil
- indicate the direction of \vec{E}_{nc} at the "x" locations outside the coil

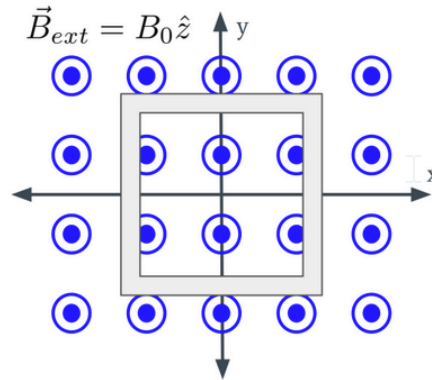
- B. [2 pts] Viewed from the other side of the loop (the side opposite the bar magnet), does the induced current run clockwise or counterclockwise in the circular coil?

- C. [3 pts] In a second experiment, a bar magnet is placed so that axis of the bar magnet is perpendicular to the axis of the coil. The magnet is then moved away from the coil along the axis of the coil as shown in the diagram. Viewed from the other side of the loop (the side opposite the bar magnet), describe the induced current in the circular coil.



FARADAY'S LAW 2

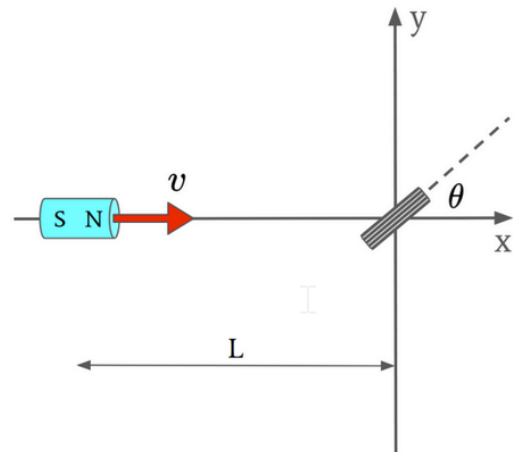
Consider a square loop of metal wire. The wire initially has sides of length ℓ_0 and is oriented so that its surface normal points in the positive z -direction as seen in the diagram. Everywhere in space is a uniform, constant, external magnetic field $\vec{B}_{ext} = B_0\hat{z}$ where B_0 is a positive constant.



- A. [5 pts] Which of the following will generate an emf around the loop (circle all that apply)?
- Moving the loop in one of the following directions: x-direction, y-direction, z-direction
 - Stretching the loop while keeping the area fixed
 - Stretching the loop so that the area is changing
 - Rotating the loop out of its initial plane
 - Moving a permanent magnetic through the loop
- B. [10 pts] Suddenly the width and length of the loop begin to change so that the area of the loop at time t is given by $A(t) = \ell_0^2(1 + t/T)(1 - t/T)$. Calculate the magnitude of the emf around the loop at time t such that $0 < t < T$.
- C. [5 pts] What is the direction of the current around the loop, when viewed from the positive z -axis, at time t such that $0 < t < T$, based on the emf you calculated in the previous question (circle all that apply)?
- Clockwise
 - Counterclockwise
 - Alternating between Clockwise and Counterclockwise
 - There is no current
 - Not enough information is given to answer this question.
- D. [5 pts] Consider the magnetic field at time t , such that $0 < t < T$, created by the current going around the loop found in the previous question. What effect does this magnetic field have on the total magnetic flux through the loop (i.e. the flux from the field produced by the current plus the external field)? The area defined by the loop has a normal vector pointing in the positive z -direction. (circle all that apply)
- The current loop increases the total magnetic flux
 - The current loop decreases the total magnetic flux
 - The current loop alternates between increasing and decreasing the total magnetic flux
 - The current loop does not change the total magnetic flux
 - Not enough information is given to answer this question.

FARADAY'S LAW 3

A bar magnet is moving at a constant speed velocity $\vec{v} = \langle v_0, 0, 0 \rangle$ along the x-axis towards the origin. At a particular instant in time, the magnet is located a distance L from the origin and has a dipole moment μ_{bar} . Located at the origin is a small metal coil made of N circular loops of radius R . The coil is tilted so that it makes an angle of θ with the positive x-axis as indicated in the diagram. If θ were exactly 90 degrees, for example, the loops of the coil would all lie in the yz-plane and the magnet would eventually pass right through the center of the coil. In the following questions you can assume that the length $L \gg R$ and that the magnet is far from the origin so that the field is approximately constant over the area of the loops.

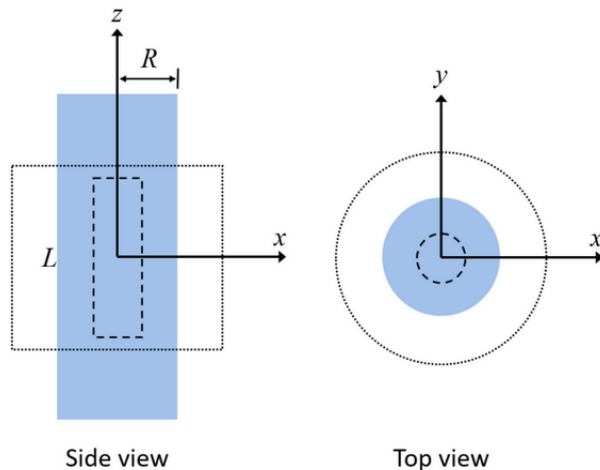


- [5 pts] Calculate the magnetic field at the center of the coil.
- [10 pts] Determine the magnitude of the magnetic flux through the coil.
- [10 pts] Calculate the magnitude of the $\mathcal{E}mf$ induced in the coil.
- [Extra Credit: 5 pts] Would the induced current in the coil repel or attract the bar magnet? Briefly explain how you determined this.

GAUSS'S LAW

GAUSS'S LAW 1

A long solid plastic cylinder of radius R and height L is bombarded by an electron beam, and gains a charge of $-q$ distributed uniformly throughout its volume. In the following questions you will need to consider two Gaussian surfaces to calculate the spatial electric field distribution $\vec{E}(\vec{r})$. Region (i) is the volume enclosed by the cylindrical surface with radius $r \leq R$ and height L_1 and is indicated on the diagram by a thick dashed line. Region (ii) is the volume enclosed by the cylindrical surface with radius $r \geq R$ and height L_2 and is indicated on the diagram by a thin dashed line.



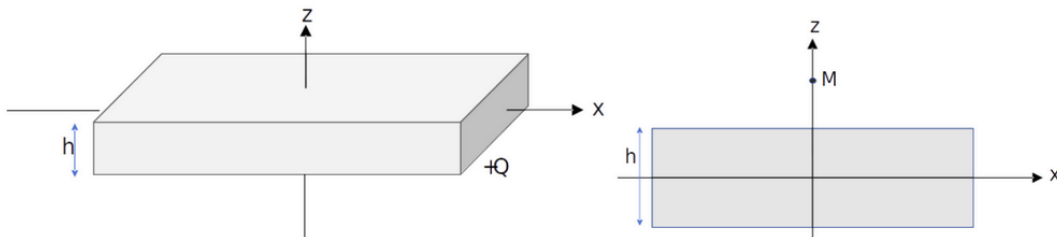
- [5 pts] Calculate the total charge enclosed in region (i)
- [6 pts] Find magnitude of the electric field $\vec{E}_{in}(\vec{r})$ at the curved surface of region (i) and describe the direction of the field on that surface.
- [5 pts] Calculate the total charge enclosed in region (ii)

D. [6 pts] Find the magnitude of the electric field $\vec{E}_{out}(\vec{r})$ at the curved surface of region (ii) and describe the direction of the field on that surface.

E. [3 pts] Now suppose that we paint the sidewall of the cylinder uniformly with a thin layer of positive charge $+q$ so that, as a whole, the cylinder is neutral. Recalculate the electric field $\vec{E}_{in}(\vec{r})$ in region (i).

GAUSS'S LAW 2

Consider a square slab of plastic with height h and area L^2 that is uniformly charged everywhere with charge $+Q$ centered on the origin. The diagram is not drawn to scale and the slab is much larger in the x and y directions so that $h \ll L$. In the following questions you will use Gauss's Law to calculate the magnitude of the electric field at different locations. By symmetry, the direction of the field everywhere should only point in the positive or negative z direction.



A. [2 pts] On the xz -plane view of the slab sketch the Gaussian surface that you would use to find the electric field at point $\vec{M} = \langle 0, 0, z_M \rangle$ located outside of the slab.

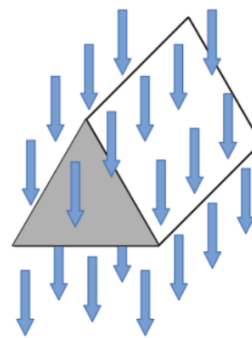
B. [3 pts] Calculate the total charged enclosed with the Gaussian surface you chose in the previous question. Be sure define and variables you created to describe the size of your surface.

C. [10 pts] Determine the magnitude of the electric field $|\vec{E}_M|$ at location \vec{M}

D. [10 pts] Determine the magnitude of the electric field $|\vec{E}_P|$ at point $\vec{P} = \langle 0, 0, z_P \rangle$ located inside the slab (i.e. $|z_P| < h/2$). Drawing a diagram would be helpful for both you and the grader!

GAUSS'S LAW 3

Two equilateral triangles (each with sides of length d) are connected to three identical rectangles (each with sides of length d and L). these five surfaces form a tent which completely encloses the volume indicated in the diagram. At every point in the space, there is a constant, downward-pointing electric field with magnitude E .

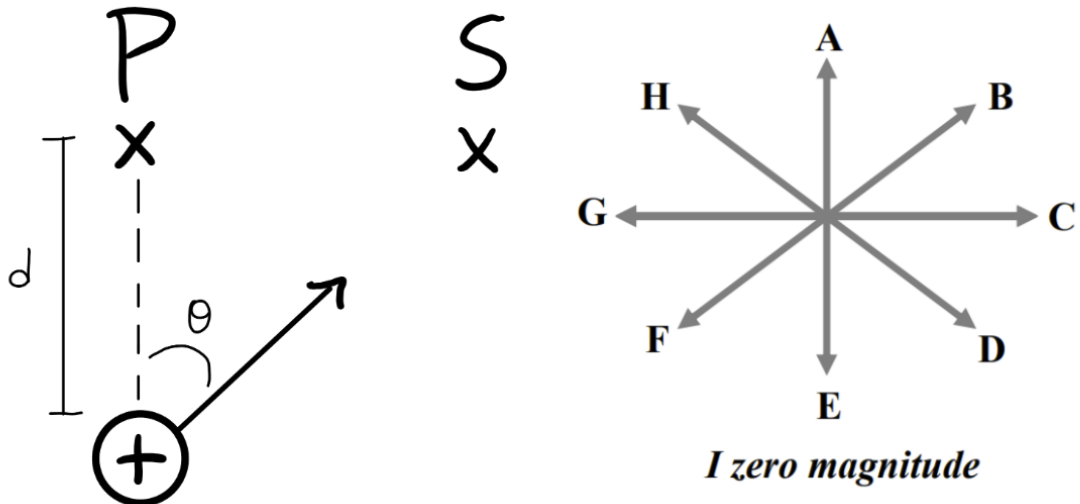


- Calculate the electric flux through the front surface of the tent (the shaded triangle). Show your work here and in all subsequent parts.
- Calculate the electric flux through the back surface of the tent (the triangle not seen in the diagram).
- Calculate the electric flux through the top right rectangular surface of the tent (the white rectangle seen in the diagram).
- Calculate the electric flux through the bottom rectangular surface of the tent (not seen in the diagram but connected to the flat bottom of both triangles).
- Calculate the total charge enclosed in the tent. To earn credit, you must show how you determined this.

ELECTROMAGNETIC WAVES

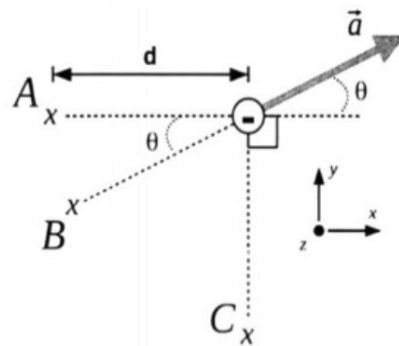
ELECTROMAGNETIC WAVES 1

A proton is accelerated in the direction shown. At observation points P and S, what are the directions of \vec{a}_\perp , $\vec{v}_{\text{radiation}}$? Find the magnitude and direction of the radiative electric and magnetic fields at point P.



ELECTROMAGNETIC WAVES 2

Beginning at time $t = 0$ an electron accelerates very briefly in the direction shown. The magnitude of this acceleration is a . Detectors of electric and magnetic fields are placed at locations A, B, and C, all a distance d from the charge. As shown in the diagram, the acceleration vector makes an angle θ with the x axis.



A. What can we say about the radiative MAGNETIC field at location A? (select one)

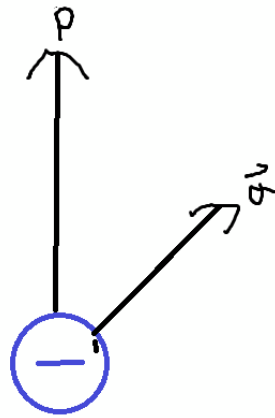
- The radiative magnetic field points in the opposite direction from the radiative electric field
- The radiative magnetic field points into the page
- The radiative magnetic field points out of the page

B. What is the magnitude of the radiative magnetic field at location A?

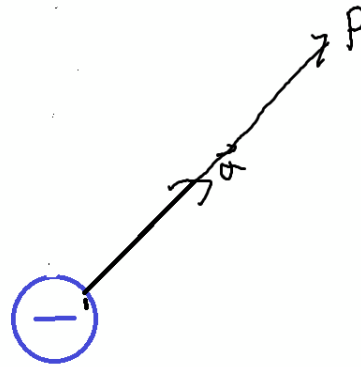
ELECTROMAGNETIC WAVES 3

For each of the following, give the direction of the radiative electric field at location P.

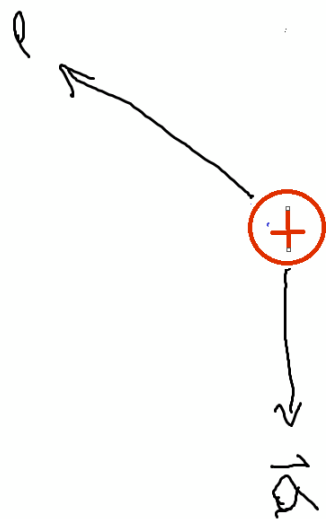
- A



- B



- C



KEYS

ELECTRIC FIELDS AND FORCES

ELECTRIC FIELDS AND FORCES 1

(a) $\vec{F} = \frac{1}{4\pi\epsilon_0} \cdot \frac{2q^2s}{d^3} \hat{x}$

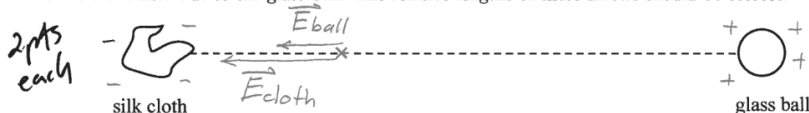
(b) $\frac{-1}{4\pi\epsilon_0} \frac{q}{d^2} \left(\frac{s}{4d} + 1 \right) \hat{x}$

(c) separation, $s_0 = \frac{\alpha}{2} \cdot \frac{1}{4\pi\epsilon_0} \frac{1}{d^2} \left(\frac{s}{4d} + 1 \right)$

ELECTRIC FIELDS AND FORCES 2

A small glass ball is rubbed all over with a small silk cloth and acquires a charge of +5 nC. The silk cloth and the glass ball are placed 30 cm apart. *If any of the electric fields you are asked to draw is zero, state this explicitly.*

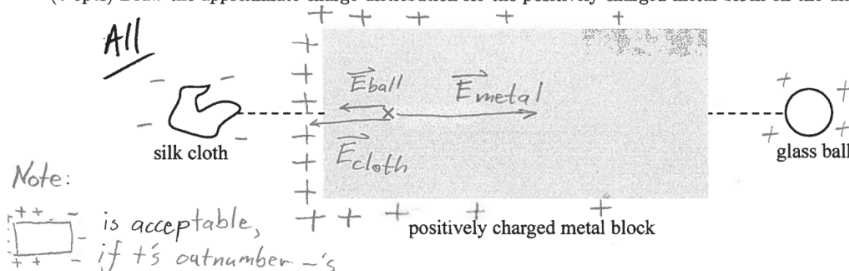
(a 4pts) At the location marked "x," draw and label two arrows, representing the electric field due to the silk cloth and the electric field due to the glass ball. The relative lengths of these arrows should be correct.



Note: Students do not need to draw charges on objects

Now a *positively charged* metal block is placed between the two objects.

(b 5pts) Draw the approximate charge distribution for the positively charged metal block on the diagram below.

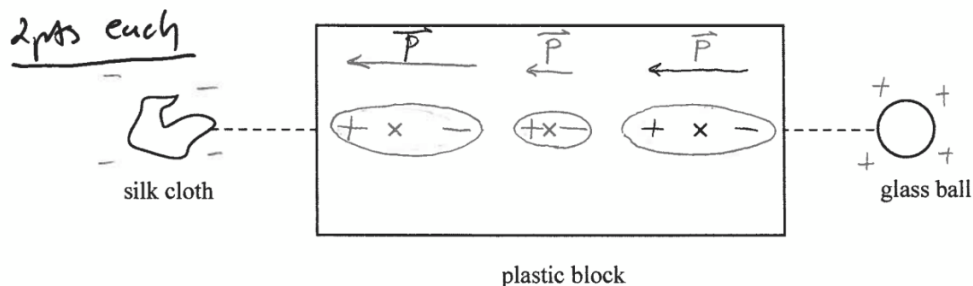


(c 10pts) At the location marked "x" draw and label four arrows, using the same scale as in part (a):

- The electric field due to the silk cloth \vec{E}_{cloth}
 - The electric field due to the glass ball \vec{E}_{ball}
 - The electric field due to the charges in and/or on the metal block \vec{E}_{metal}
 - The net electric field \vec{E}_{net}
- 3pts each*
 $E_{net} = 0$ 1pt

Now the metal block is removed, and replaced by a neutral plastic block.

(g 6pts) Show the polarization of a molecule at each location marked "x" below.



ELECTRIC POTENTIAL

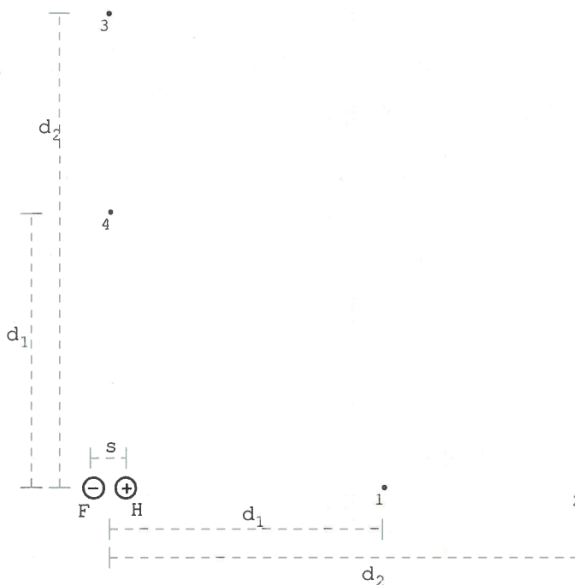
ELECTRIC POTENTIAL 1

$$\frac{2Q}{4\pi\epsilon_0} \left(\frac{-1}{R+d} + \frac{1}{R+d+L} \right)$$

ELECTRIC POTENTIAL 2

An HF molecule is composed of two point charges, H^+ and F^- , with charges $+e$ and $-e$ respectively separated by a distance s (i.e. and electric dipole).

1. (40 points) Calculate the potential difference $V_2 - V_1$, where locations 1 and 2 are shown on the diagram. The distances d_1 and d_2 are much larger than the internuclear separation s . Make sure the sign of your answer is correct. Show all of your work.



$$V_2 - V_1 = - \int_{\text{1}}^{\text{2}} \vec{E}(\vec{r}) \cdot d\vec{P} = - \int_{\text{1}}^{\text{2}} (E_{\text{axis}} \hat{x}) \cdot (dx \hat{x})$$

$$= - \int_{d_1}^{d_2} \frac{1}{4\pi\epsilon_0} \frac{2es}{x^3} dx = \frac{-2es}{4\pi\epsilon_0} \left(\frac{1}{-2} \frac{1}{x^2} \right)_{d_1}^{d_2}$$

$\int x^n = \frac{1}{n+1} x^{n+1}$

$$V_2 - V_1 = \frac{es}{4\pi\epsilon_0} \left(\frac{1}{d_2^2} - \frac{1}{d_1^2} \right)$$

Grading rubric

- 2 pts Clerical
- 8 pts Minor
- 16 pts Major
- 32 pts BTW

2. (20 points) A proton is released from rest at location 1. Calculate the speed of the proton when it is very far (i.e. $r = \infty$) from the dipole. Show all of your work.

Energy principle: $\Delta K + q \Delta V = 0$ with $q = e$ and:

$$\star \Delta V = V_{\infty, \text{axis}} - V_1 = \boxed{A} = - \int_1^{\infty} E_{\text{axis}} dx = \frac{es}{4\pi\epsilon_0} \left(\frac{1}{x^2} \right)_{d_1}^{\infty} = \frac{-es}{4\pi\epsilon_0 d_1^2}$$

$$\star \Delta K = K_{\infty} - K_1 = \frac{1}{2} m v_{\infty}^2$$

$$\rightarrow \frac{1}{2} m v_{\infty}^2 - e \frac{es}{4\pi\epsilon_0 d_1^2} = 0 \rightarrow \boxed{v_{\infty} = \sqrt{\frac{5}{2\pi\epsilon_0 m} \left(\frac{e}{d_1} \right)}}$$

3. (20 points) What is the potential difference $V_4 - V_3$? Locations 3 and 4 are on a line extending through the midpoint of the molecule. Show all of your work.

$$V_4 - V_3 = - \int_{\text{③}}^{\text{④}} \vec{E}(\vec{r}) \cdot d\vec{l} = - \int_{\text{③}}^{\text{④}} (-E_{\perp \text{axis}} \hat{x}) \cdot (dy \hat{y})$$

③ \vec{E} points to the left $d\vec{l}$ points upward

$$\rightarrow \boxed{V_4 - V_3 = 0}$$

Grading rubric for questions 2, 3 and 4

- 1 pt Clerical
- 4 pts Minor
- 8 pts Major
- 16 pts BTV

4. (20 points) Calculate the potential difference $V_1 - V_4$, where locations 1 and 4 are shown on the diagram.

$V_1 - V_4 = \underbrace{(V_1 - V_{\infty, \text{axis}})}_{\boxed{A}} + \underbrace{(V_{\infty, \text{axis}} - V_{\infty, \perp \text{axis}})}_{\boxed{B}} + \underbrace{(V_{\infty, \perp \text{axis}} - V_4)}_{\boxed{C}}$

$\star \boxed{B} = 0$ because $V(\infty) = 0$

$\star \boxed{C} = 0$ because of question 3)

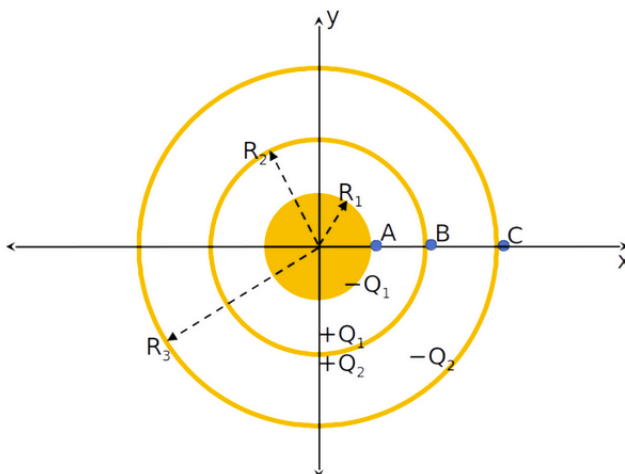
$$\rightarrow \boxed{V_1 - V_4 = \frac{es}{4\pi\epsilon_0 d_1^2}}$$

ELECTRIC POTENTIAL 3

As shown in the figure, a solid metal sphere of radius R_1 and charge $-Q_1$ is surrounded by two concentric spherical metal shells with radii R_2 and R_3 , respectively. The charges on these concentric spherical metal shells are:

- (i) $+Q_1$ on the inner surface of R_2 ,
- (ii) $+Q_2$ on the outer surface of R_2 ,
- (iii) $-Q_2$ on the inner surface of R_3 ,
- (iv) no charge on the outer surface of R_3 .

All charges are distributed uniformly across inner and outer surfaces.



- A. [3 pts] At point C just outside the spherical shell R_3 , what is the electric potential V_C (that is, calculate $V_C - V_\infty$)? Note that when calculating the potential difference near infinitesimally thin shells, it does not matter if one of the end points is on or just outside the surface of the shell.

$$V_C - V_\infty = \text{potential if all sources act as point-charges located at the origin}$$

$$= \frac{1}{4\pi\epsilon_0} \frac{-Q_1}{R_3} + \frac{1}{4\pi\epsilon_0} \frac{Q_1}{R_3} + \frac{1}{4\pi\epsilon_0} \frac{-Q_2}{R_3} + \frac{1}{4\pi\epsilon_0} \frac{Q_2}{R_3}$$

→ $V_C = 0$ All or nothing

- B. [10 pts] Point A is just outside the metal sphere of radius R_1 . What is the potential difference, $V_A - V_C$? Show your work.

$$V_A - V_C = (V_A - V_B) + (V_B - V_C)$$

potential due to "point charge" $-Q_1$
potential due to "point charge" $+Q_2$

$$= \frac{-Q_1}{4\pi\epsilon_0} \left(\frac{1}{R_1} - \frac{1}{R_2} \right) + \frac{Q_2}{4\pi\epsilon_0} \left(\frac{1}{R_2} - \frac{1}{R_3} \right)$$

Grading Rubric

- 1 pt Correct
- 2 pts Minor
- 4 pts Major
- 8 pts BTW

$$\rightarrow V_A - V_C = \frac{1}{4\pi\epsilon_0} \left(\frac{-Q_1}{R_1} + \frac{Q_1}{R_2} + \frac{Q_2}{R_2} - \frac{Q_2}{R_3} \right)$$

- C. [8 pts] What is the electric potential V_0 at the center of the metal sphere (that is, calculate $V_0 - V_\infty$)? Hint: You should use your results from parts A and B and think about how the charge would be distributed over the metal sphere.

$$V_0 - V_\infty = (V_0 - V_A) + (V_A - V_C) + (V_C - V_\infty)$$

$$* V_0 - V_A = - \int_A^0 \vec{E}_{\text{inside metal sphere}} \cdot d\vec{l}$$

But since we're inside a conductor with charges on the surface only

$$\rightarrow \vec{E}_{\text{inside}} = 0$$

$$\rightarrow V_0 - V_A = 0$$

$$\rightarrow V_0 - V_\infty = 0 + \frac{1}{4\pi\epsilon_0} \left(\frac{-Q_1}{R_1} + \frac{Q_1}{R_2} + \frac{Q_2}{R_2} - \frac{Q_2}{R_3} \right) + 0$$

$$\rightarrow V_0 = \frac{1}{4\pi\epsilon_0} \left(\frac{-Q_1}{R_1} + \frac{Q_1}{R_2} + \frac{Q_2}{R_2} - \frac{Q_2}{R_3} \right)$$

Watch for
POE!

Grading
rubric:
-1 pt/Correct
-2 pts/Minor
-3 pts/Major
-6 pts/STW

- D. [4 pts] Suppose that $V_A = V_C$, as would be the case when the two components are connected by a metal wire. In this case, what is the ratio Q_1/Q_2 ?

$$\text{If } V_A = V_C \rightarrow Q_1 \left(\frac{-1}{R_1} + \frac{1}{R_2} \right) + Q_2 \left(\frac{1}{R_2} - \frac{1}{R_3} \right) = 0$$

$$\rightarrow \frac{Q_1}{Q_2} = \frac{-\left(\frac{1}{R_2} - \frac{1}{R_3} \right)}{\left(\frac{-1}{R_1} + \frac{1}{R_2} \right)} = \frac{-\left(\frac{R_3 - R_2}{R_2 R_3} \right)}{\left(\frac{-R_2 + R_1}{R_1 R_2} \right)}$$

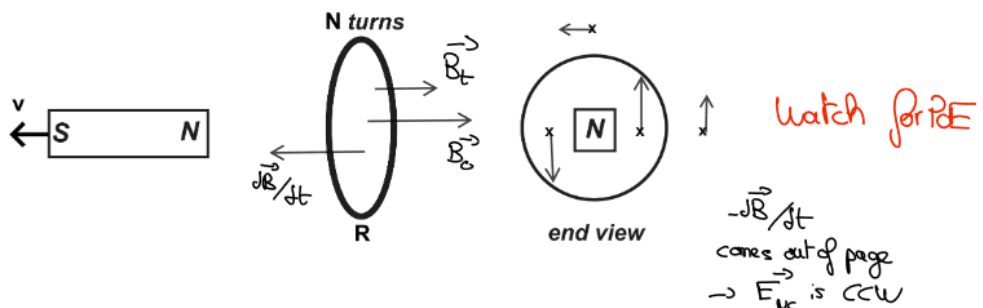
$$\rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_3} \frac{R_2 - R_3}{R_1 - R_2}$$

Watch for P&E
All or nothing

FARADAY'S LAW

FARADAY'S LAW 1

In one experiment, the north pole of a bar magnet points toward a thin circular coil of wire containing N turns of radius R . The magnet is then moved away from the coil along the axis of the coil as shown in the diagram. In the following questions you should assume that, at any given time, the magnetic field produced by the magnet is uniform over the area enclosed by the loop.



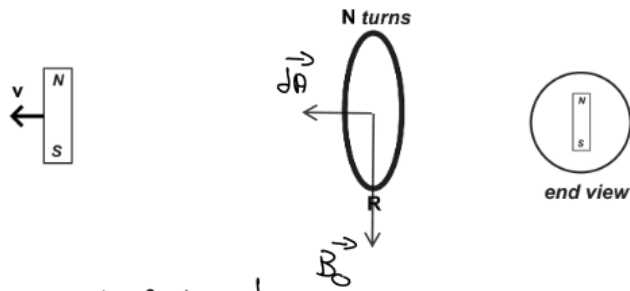
A. [20 pts] Draw and label the following arrows (relative lengths of the arrows should scale):

- +4 pts • \vec{B}_0 the magnetic field at the center of the coil before the magnet was moving
 - +4 pts • \vec{B}_t the magnetic field at the center of the coil while the magnet is moving
 - +4 pts • $\frac{d\vec{B}}{dt}$ the change in the magnetic field as the magnet is moving
 - +4 pts • indicate the direction of \vec{E}_{nc} at the "x" locations inside the coil
 - +4 pts • indicate the direction of \vec{E}_{nc} at the "x" locations outside the coil
- } grade 2 pts for direction
} 2 pts for magnitude
} 2 pt/arrow's direction
} (do not grade magnitude)

B. [2 pts] Viewed from the other side of the loop (the side opposite the bar magnet), does the induced current run clockwise or counterclockwise in the circular coil?

CCW
(same direction as \vec{E}_{nc})
All or nothing (watch for P&E)

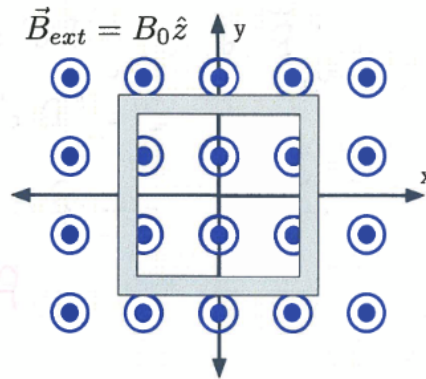
C. [3 pts] In a second experiment, a bar magnet is placed so that axis of the bar magnet is perpendicular to the axis of the coil. The magnet is then moved away from the coil along the axis of the coil as shown in the diagram. Viewed from the other side of the loop (the side opposite the bar magnet), describe the induced current in the circular coil.



Since $\vec{B}_0 \perp \vec{A} \rightarrow \Phi = 0$ at all time!
 \rightarrow no induced current *All or nothing*

FARADAY'S LAW 2

Consider a square loop of metal wire. The wire initially has sides of length ℓ_0 and is oriented so that its surface normal points in the positive z-direction as seen in the diagram. Everywhere in space is a uniform, constant, external magnetic field $\vec{B}_{ext} = B_0 \hat{z}$ where B_0 is a positive constant.



A. [5 pts] Which of the following will generate an emf around the loop (circle all that apply)?

- Moving the loop in one of the following directions: x-direction, y-direction, z-direction
- Stretching the loop while keeping the area fixed
- Stretching the loop so that the area is shrinking
- Rotating the loop out of its initial plane
- Moving a permanent magnet through the loop

+ 2 pts for each wrong answer
+ 2 pts for 1 good answer
+ 4 pts for 2 good answers
+ 5 pts for 3 good answers

- B. [10 pts] Suddenly the width and length of the loop begin to change so that the area of the loop at time t is given by $A(t) = \ell_0^2(1 + t/T)(1 - t/T)$. Calculate the magnitude of the emf around the loop at time t such that $0 < t < T$.

$$\Phi(t) \stackrel{(1)}{=} \vec{B}_{\text{ext}} \cdot \vec{A}(t) = B_0 A(t) \quad \parallel \quad |\text{Emf}| \stackrel{(2)}{=} \left| \frac{d\Phi}{dt} \right|$$

$$\begin{aligned} (1) \quad \frac{d\Phi(t)}{dt} &= B_0 \ell_0^2 \frac{d}{dt} \left(\left(1 + \frac{t}{T}\right) \left(1 - \frac{t}{T}\right) \right) \\ &= B_0 \ell_0^2 \left(\left(1 - \frac{t}{T}\right) \frac{1}{T} + \left(1 + \frac{t}{T}\right) \left(-\frac{1}{T}\right) \right) \\ &= \frac{-2 B_0 \ell_0^2 t}{T^2} \end{aligned}$$

$$(2) \quad |\text{Emf}| = \frac{2 B_0 \ell_0^2 t}{T^2}$$

Grading rubric: -1 pt / Correct -4 pts / Major
-2 pts / Minor -8 pts / BTV

- C. [5 pts] What is the direction of the current around the loop, when viewed from the positive z -axis, at time t such that $0 < t < T$, based on the emf you calculated in the previous question (circle all that apply)?

- Clockwise
- Counterclockwise
- Alternating between Clockwise and Counterclockwise
- There is no current
- Not enough information is given to answer this question.

Flux is decreasing
Normal to loop is \hat{z}
→ current generates field in \hat{z} direction

- D. [5 pts] Consider the magnetic field at time t , such that $0 < t < T$, created by the current going around the loop found in the previous question. What effect does this magnetic field have on the total magnetic flux through the loop (i.e. the flux from the field produced by the current plus the external field)? The area defined by the loop has a normal vector pointing in the positive z -direction. (circle all that apply)

- The current loop increases the total magnetic flux
- The current loop decreases the total magnetic flux
- The current loop alternates between increasing and decreasing the total magnetic flux
- The current loop does not change the total magnetic flux
- Not enough information is given to answer this question.

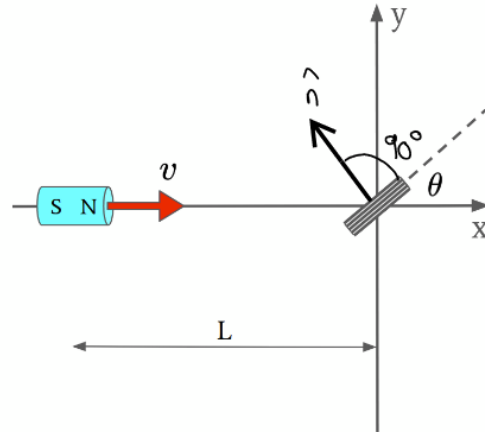
Since I generates a field that points in \hat{z}

Question c)
- watch for PoE
- All or nothing

Question d)
- watch for PoE (from question C)
- All or nothing

FARADAY'S LAW 3

A bar magnet is moving at a constant speed velocity $\vec{v} = \langle v_0, 0, 0 \rangle$ along the x-axis towards the origin. At a particular instant in time, the magnet is located a distance L from the origin and has a dipole moment μ_{bar} . Located at the origin is a small metal coil made of N circular loops of radius R . The coil is tilted so that it makes an angle of θ with the positive x-axis as indicated in the diagram. If θ were exactly 90 degrees, for example, the loops of the coil would all lie in the yz-plane and the magnet would eventually pass right through the center of the coil. In the following questions you can assume that the length $L \gg R$ and that the magnet is far from the origin so that the field is approximately constant over the area of the loops.



A. [5 pts] Calculate the magnetic field at the center of the coil.

$$\vec{B}_{\text{magnet}} = \text{field of dipole on axis}$$

$$\rightarrow \vec{B}_m = \frac{\mu_0}{4\pi} \frac{2\mu_{\text{bar}}}{L^3} \hat{x}$$

. All or nothing
 . -1 pt for factor of 2
 . -1 pt for missing vector

watch for PoE

B. [10 pts] Determine the magnitude of the magnetic flux through the coil.

$$|\Phi| = N |\vec{B}_m \cdot \hat{n} A| = N \times \underset{\substack{\uparrow \\ \text{loops}}}{A} \times \underset{\substack{\uparrow \\ \text{area of loops}}}{B_m} \times \underset{\substack{\uparrow \\ \text{check figure}}}{|\hat{x} \cdot \hat{n}|}$$

$$= N \times \pi R^2 \times \frac{\mu_0 2\mu}{4\pi L^3} \times |\cos(90^\circ + \theta)|$$

$$\rightarrow |\Phi| = \frac{\mu_0}{4\pi} \frac{2\mu_{\text{bar}} N \pi R^2}{L^3} \sin \theta$$

Grading Rubric
 -1 pt: Period
 -2 pts: Minor
 -4 pts: Major
 -8 pts: BTV

watch for PoE
 wrong trig: minor

C. [10 pts] Calculate the magnitude of the \mathcal{E}_{mf} induced in the coil.

$$\mathcal{E}_{mf} = \left| \frac{d\Phi}{dt} \right| = \frac{\mu_0}{4\pi} 2\mu_{\text{bar}} N^2 R^2 \sin\theta \left| \frac{d}{dt} \left(\frac{1}{L^3} \right) \right|$$

$$\frac{d}{dt} \left(\frac{1}{L^3} \right) = \frac{d}{dL} \left(\frac{1}{L^3} \right) \times \frac{dL}{dt} = \frac{-3}{L^4} \times -v$$

$$\rightarrow \mathcal{E}_{mf} = \frac{\mu_0}{4\pi} \frac{6v\mu_{\text{bar}} N^2 R^2 \sin\theta}{L^4}$$

Grading
Rubric

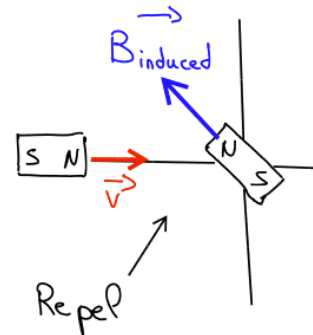
- 1 pt: Derive
- 2 pts: Minor
- 4 pts: Major
- 8 pts: BTW

Watch for P&E

D. [Extra Credit: 5 pts] Would the induced current in the coil repel or attract the bar magnet? Briefly explain how you determined this.

Answer: **Repel**. The induced current winds such that it creates a field that points in the \hat{n} -direction. If we think of the coil as a bar magnet, it is oriented:

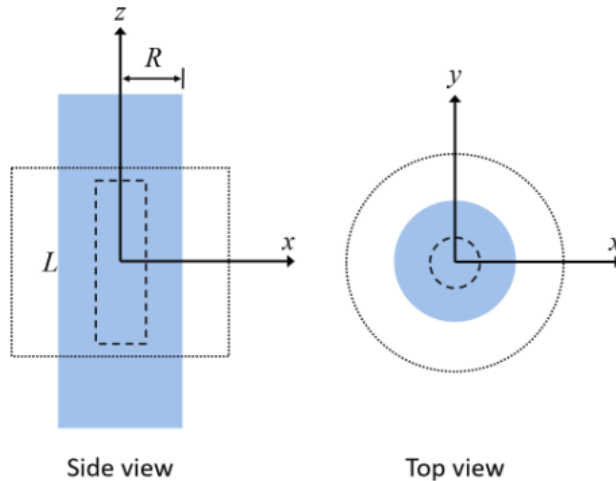
All or nothing



GAUSS'S LAW

GAUSS'S LAW 1

A long solid plastic cylinder of radius R and height L is bombarded by an electron beam, and gains a charge of $-q$ distributed uniformly throughout its volume. In the following questions you will need to consider two Gaussian surfaces to calculate the spatial electric field distribution $\vec{E}(\vec{r})$. Region (i) is the volume enclosed by the cylindrical surface with radius $r \leq R$ and height L_1 and is indicated on the diagram by a thick dashed line. Region (ii) is the volume enclosed by the cylindrical surface with radius $r \geq R$ and height L_2 and is indicated on the diagram by a thin dashed line.



A. [5 pts] Calculate the total charge enclosed in region (i)

$Q_{enc} = \rho \times V_{enc}$ with:

$\rho \equiv \frac{\text{charge}}{\text{density}} = \frac{\text{total charge}}{\text{total volume}} = \frac{-q}{\pi R^2 L}$ and $V_{enc} = \pi r^2 L_1$

$\rightarrow Q_{enc} = -q \left(\frac{r}{R}\right)^2 \frac{L_1}{L}$ All or nothing

B. [6 pts] Find magnitude of the electric field $\vec{E}_{in}(\vec{r})$ at the curved surface of region (i) and describe the direction of the field on that surface.

Gauss Law: $\oint \vec{E}_{in} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_0}$ ← surface Area of cylinder

Since the field points inward: $-|\vec{E}_{in}| 2\pi r L_1 = \frac{-q}{\epsilon_0} \frac{r^2}{R^2} \frac{L_1}{L}$ (+2pts)

$\rightarrow |\vec{E}_{in}| = \frac{q r}{2\pi \epsilon_0 R^2 L}$ (+2pts) (Take -1 pt if negative)

Points inward! (+2pts) (watch for PoE)

C. [5 pts] Calculate the total charge enclosed in region (ii)

$$Q_{\text{enc}} \equiv \rho \times \text{Volume enclosed by dashed line and the solid plastic cylinder}$$

$$= \frac{-q}{\pi R^2 L} \times \pi R^2 L_2$$

$$\rightarrow Q_{\text{enc}} = -q \frac{L_2}{L} \quad \text{All or nothing}$$

D. [6 pts] Find the magnitude of the electric field $\vec{E}_{\text{out}}(\vec{r})$ at the curved surface of region (ii) and describe the direction of the field on that surface.

Gauss Law: $\oint \vec{E}_{\text{out}} \cdot d\vec{A} = \frac{Q_{\text{enc}}}{\epsilon_0}$

Field still points inward $- |\vec{E}_{\text{out}}| \times 2\pi r L_2 = \frac{-q}{\epsilon_0} \frac{L_2}{L} \quad (+2\text{pts})$

$$\rightarrow |\vec{E}_{\text{out}}| = \frac{q}{2\pi \epsilon_0 L r} \quad (+2\text{pts})$$

Field points inward $(+2\text{pts}) \quad (\text{watch for POE})$

E. [3 pts] Now suppose that we paint the sidewall of the cylinder uniformly with a thin layer of positive charge $+q$ so that, as a whole, the cylinder is neutral. Recalculate the electric field $\vec{E}_{\text{in}}(\vec{r})$ in region (i).

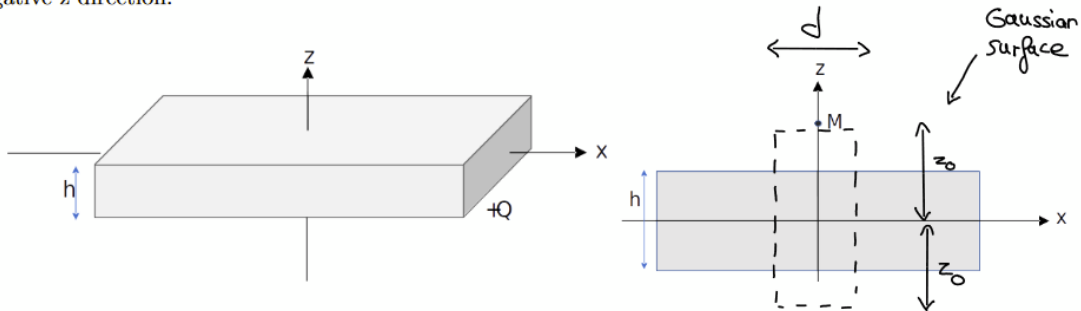
Since the layer of positive charge is outside of the inner Gaussian surface, it does not affect the value of the E-field:

$$\vec{E}_{\text{in}} \equiv \text{Same as question} \quad \boxed{B}$$

All or nothing
(watch for POE)

GAUSS'S LAW 2

Consider a square slab of plastic with height h and area L^2 that is uniformly charged everywhere with charge $+Q$ centered on the origin. The diagram is not drawn to scale and the slab is much larger in the x and y directions so that $h \ll L$. In the following questions you will use Gauss's Law to calculate the magnitude of the electric field at different locations. By symmetry, the direction of the field everywhere should only point in the positive or negative z direction.



A. [2 pts] On the xz -plane view of the slab sketch the Gaussian surface that you would use to find the electric field at point $\vec{M} = \langle z_0, 0, 0 \rangle$ located outside of the slab. **ARE or nothing**

B. [3 pts] Calculate the total charged enclosed with the Gaussian surface you chose in the previous question. Be sure define and variables you created to describe the size of your surface.

charge density = $\rho = \frac{Q}{\text{total volume}} = \frac{Q}{hL^2}$. +1 pt for charge density
 . +2 pts for Q_{enc}

$Q_{\text{enc}} = \rho \times \text{volume of charges enclosed}$
 $= \frac{Q}{hL^2} \times h d^2$
 $\rightarrow Q_{\text{enc}} = Q \frac{d^2}{L^2}$

. Watch for Gaussian surface. Other surfaces like one enclosing all charges ($2z_0 \times L$) is acceptable.

C. [10 pts] Determine the magnitude of the electric field $|\vec{E}_M|$ at location \vec{M}

$\oiint \vec{E} \cdot \hat{n} dA = \frac{Q_{\text{enc}}}{\epsilon_0}$

. LHS = contribution only from top and bottom surfaces. Sides cancel. = $\iint_{\text{top}} \vec{E} \cdot \hat{n} dA + \iint_{\text{bottom}} \vec{E} \cdot \hat{n} dA + \iint_{\text{sides}} \vec{E} \cdot \hat{n} dA$

= $2|\vec{E}_M| \times \text{area top surface}$

= $2|\vec{E}_M| d^2$

by symmetry, field at top equals field at bottom in magnitude.

$\rightarrow 2|\vec{E}_M| d^2 = \frac{Q}{\epsilon_0} \frac{d^2}{L^2}$

$\rightarrow |\vec{E}_M| = \frac{Q}{2\epsilon_0 L^2}$

watch for PE on Q_{enc} and Gaussian surface

Grading Rubric

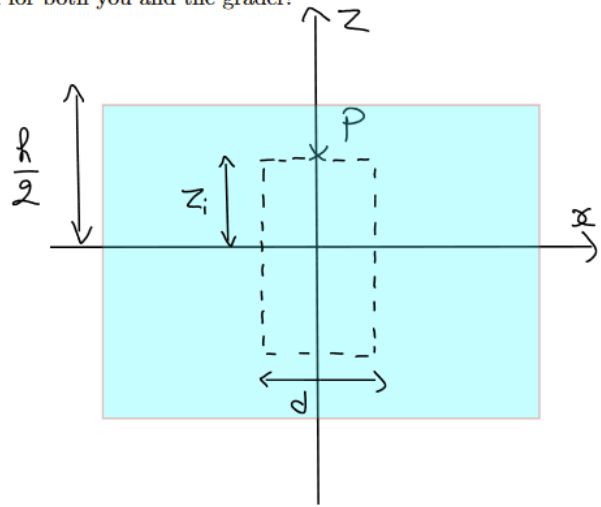
- 1 pt: Correct .4 pts: Major
- 2 pts: Minor .8 pts: BTW

- D. [10 pts] Determine the magnitude of the electric field $|\vec{E}_P|$ at point $\vec{P} = \langle 0, 0, z_i \rangle$ located inside the slab (i.e. $|z_i| < h/2$). Drawing a diagram would be helpful for both you and the grader!

$$\oint \vec{E} \cdot \hat{n} dA = \frac{Q_{enc}}{\epsilon_0}$$

- The Left Hand Side remains the same as in question C:

$$LHS = 2 |\vec{E}_P| d^2$$



- The Right Hand Side changes. we still need the charge density

$$\rho = \frac{\text{total charge}}{\text{total volume}} = \frac{Q}{RL^2}$$

$$\begin{aligned} \rightarrow Q_{enc} &= \rho V_{enclosed} \\ &= \frac{Q}{RL^2} \times 2z_i d^2 \\ &= Q \frac{2z_i}{R} \frac{d^2}{L^2} \end{aligned}$$

$$\rightarrow 2 |\vec{E}_P| d^2 = \frac{2z_i Q d^2}{R \epsilon_0 L^2}$$

$$\rightarrow |\vec{E}_P| = \frac{Q}{\epsilon_0 L^2} \frac{z_i}{R}$$

Grading
Rubric

- 1 pt: Cerial
- 2 pts: Minor
- 4 pts: Major
- 8 pts: BTW

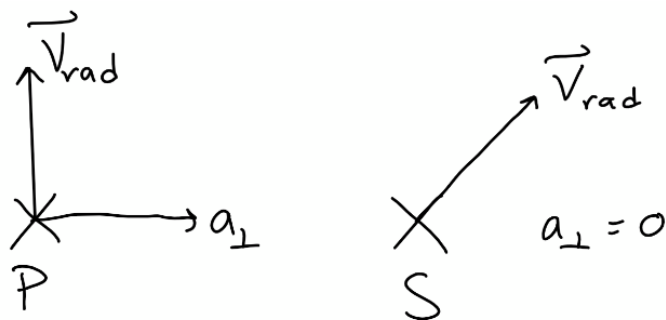
Watch for P/E

GAUSS'S LAW 3

(a) 0. (b) 0. (c) $-\frac{1}{2}EdL$. (d) EdL . (e) 0.

ELECTROMAGNETIC WAVES

ELECTROMAGNETIC WAVES 1



$$|E_{\text{rad}}| = \frac{1}{4\pi\epsilon_0} \frac{e|\vec{a}| \sin\theta}{c^2 d} \quad \hat{x}$$

$$|B_{\text{rad}}| = \frac{1}{4\pi\epsilon_0} \frac{e|\vec{a}| \sin\theta}{c^3 d} \quad \hat{z}$$

ELECTROMAGNETIC WAVES 2

A. By $\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{-q\vec{a}_\perp}{c^2 d}$, we get $\hat{E} = \hat{a}_\perp = +\hat{y}$

Then we know $\hat{E} \times \hat{B} = \hat{v}$, where \hat{v} is the direction of propagation of the wave, $-\hat{x}$

So $+\hat{y} \times \hat{B} = -\hat{x}$, so by RHR, $\hat{B} = -\hat{z}$ (into the page)

$$B. \quad \vec{E} = \frac{1}{4\pi\epsilon_0} \frac{-q\vec{a}_\perp}{c^2 d} \quad |\vec{a}_\perp| = |\vec{a}| \sin\theta$$

$$\text{Then } |\vec{E}| = \frac{1}{4\pi\epsilon_0} \frac{ea \sin\theta}{c^2 d}$$

Then using $\frac{|\vec{E}|}{|\vec{B}|} = c$, we get

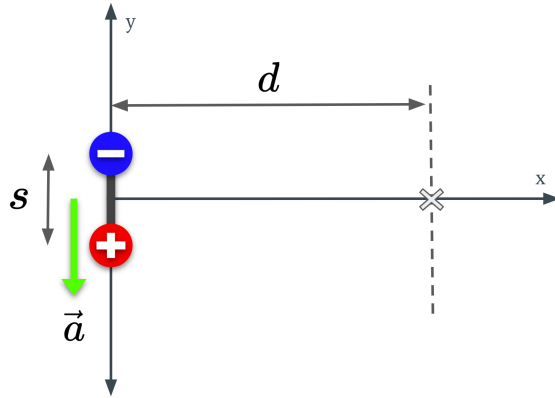
$$|\vec{B}| = \frac{1}{4\pi\epsilon_0} \frac{ea \sin\theta}{c^3 d}$$

ELECTROMAGNETIC WAVES 3

- A: To the right
- B: Electric field has magnitude zero
- C: Up and right (note that we have a positive charge, so direction is opposite of a perp)

6 The accelerating dipole

A permanent electric dipole with moment $\vec{p} = \langle 0, -qs, 0 \rangle$ is located at the origin and accelerating down with a constant acceleration $\vec{a} = \langle 0, -a_0, 0 \rangle$.



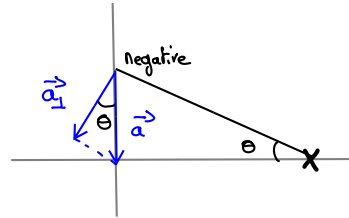
Q1: Calculate the net radiative electric and magnetic fields at observation location $\vec{r} = \langle d, 0, 0 \rangle$. Your answers should be a vector. (20 pts)

$$\vec{E}_{\text{net,rad}} = \vec{E}_{\text{m,rad}} + \vec{E}_{\text{p,rad}}$$

Negative particle:

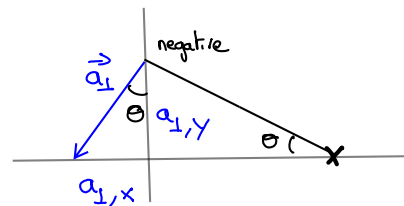
$$\vec{E}_{\text{m,rad}} = \frac{+q a_{\perp}}{4\pi\epsilon_0 c^2 r}$$

$$* r = \sqrt{\left(\frac{s}{2}\right)^2 + d^2} \quad * \cos\theta = \frac{d}{\sqrt{\left(\frac{s}{2}\right)^2 + d^2}}$$



$$* |\vec{a}_{\perp}| = a_0 \cos\theta = \frac{a_0 d}{\sqrt{s^2/4 + d^2}}$$

$$* \vec{a}_{\perp} = |\vec{a}_{\perp}| \langle -\sin\theta, -\cos\theta, 0 \rangle$$



Positive particle:

$$\vec{E}_{\text{p,rad}} = \frac{-q a_{\perp}}{4\pi\epsilon_0 c^2 r}$$

$$* \vec{a}_{\perp} = |\vec{a}_{\perp}| \langle +\sin\theta, -\cos\theta, 0 \rangle$$

Net field:

$$\vec{E}_{\text{net,rad}} = \frac{q a_0}{4\pi\epsilon_0 c^2} \left(\frac{1}{\sqrt{s^2/4 + d^2}} \right) \cos\theta \left(\langle -\sin\theta, -\cos\theta, 0 \rangle - \langle \sin\theta, -\cos\theta, 0 \rangle \right)$$

$$= \frac{q a_0}{4\pi\epsilon_0 c^2} \left(\frac{1}{\sqrt{s^2/4 + d^2}} \right) \left(\frac{d}{\sqrt{s^2/4 + d^2}} \right) \left(\frac{2 \times s/2}{\sqrt{s^2/4 + d^2}} \right) (-\hat{x})$$

$$\vec{E}_{\text{net,rad}} = \frac{-q a_0 s d}{4\pi\epsilon_0 c^2 (s^2/4 + d^2)^{3/2}} \hat{x}$$

- B-field - 1st method : $\vec{E}_{\text{net,rad}}$ and the direction of propagation are parallel

$$\rightarrow \vec{B}_{\text{net,rad}} = 0$$

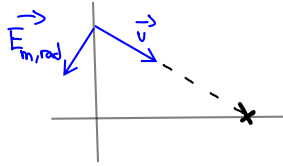
Grading Rubric for \vec{E}_{rad}

Clerical: -1 pt Major: -6 pts

Minor: -3 pts M.P.: -2 pts

Grading for B_{rad} : 5 pts (all or nothing)

- 2nd method



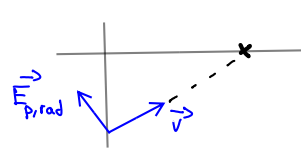
$$\rightarrow \vec{B}_{\text{m,rad}} = -|\vec{B}_{\text{m,rad}}| \hat{z}$$

$$\rightarrow \vec{B}_{\text{net,rad}} = 0$$

• minor: trig

• watch for PoE from E_{rad} to B_{rad}

• Anything involving a "retarded vector potential" is -20 pts



$$\rightarrow \vec{B}_{\text{p,rad}} = +|\vec{B}_{\text{p,rad}}| \hat{z}$$

Q2: Using the answer you found for the net radiative electric field, approximate the magnitude of the field using the binomial expansion in the limit $d \gg s$. (5 pts)

$$\begin{aligned} \text{In the limit } d \gg s : \frac{1}{(s^2/4 + d^2)^{3/2}} &= \frac{1}{d^3} \left(1 + \frac{s^2}{4d^2} \right)^{-3/2} \\ &= \frac{1}{d^3} \left(1 - \frac{3}{2} \frac{s^2}{4d^2} \right) \end{aligned}$$

$$\rightarrow \vec{E}_{\text{rad}} = \frac{-q a_0 s \hat{x}}{4\pi\epsilon_0 c^2 d^2} \left(1 - \frac{3}{8} \frac{s^2}{d^2} \right)$$

• Accept if student take $d^2 + \frac{s^2}{4} \approx d^2$

• 5/5 if correct answer

• 0 anything else

• -1 pt: missing vector