Printed name	Nine-digit GT ID

Fall 2022 IPLS I Final exam

- Print your name and nine-digit Tech ID **very neatly** in the spaces above.
- Initial the odd pages in the top margin, in case the pages of your quiz get separated during scanning.
- We will distribute the standard formula sheet separately. *Do not* turn in the formula sheet along with the test; any work on the formula sheet will not be graded.
- Read all problems carefully before attempting to solve them.
- Your work must be legible, and the organization must be clear.
- You may use any dark-colored pencil or pen that is not orange.
- You must show all work, including **correct vector notation and units** where appropriate.
- **Correct answers without adequate explanation may be marked wrong**.
- Incorrect work or explanations mixed in with correct work may be marked wrong. Cross out anything you do not want us to grade.
- Make explanations correct but brief. You do not need to write a lot.
- Include diagrams where appropriate.
- Show what goes into a calculation, not just the final number. For instance,
$$\frac{a \cdot b}{c \cdot d} = \frac{(8 \cdot 10^{-3}\ \mathrm{N})(5 \cdot 10^{6}\ \mathrm{m})}{(2 \cdot 10^{-5}\ \mathrm{N})(4 \cdot 10^{4}\ \mathrm{m})} = 5 \cdot 10^{4}$$
- Give standard SI units with your results if we don't already supply them.
- If you cannot do some portion of a problem, invent a symbol for the quantity you cannot calculate. Explain that you are doing this, and use it to do the rest of the problem.
- Some problems may include extra information (or extra variables) that are not needed for the final answer. Some problems may omit standard information that is given on the formula sheet, or common standard variables (such as g for gravity, or e for the elementary charge in electrostatics), that are necessary for their solution.
- **Simplify your answers when possible**.

Problem 1: Short answer / multiple choice

Limited or no partial credit.

[6 points] Estimate the amount of energy required to climb a flight of stairs. Keep in mind that human muscles are only about 50% efficient.

J

The average American weighs about 180 lbs = 80 kg

One story of a house is about 10 feet = 3.3 m (ceilings are 8 or 9 feet, and add another foot for the thickness of the floor)

In climbing one flight of stairs your gravitational potential energy increases by mgh = (80 kg)(10 m/s^2)(3.3m)=2640 J.

Double this to account for the 50% efficiency of your muscles, to give 5300 J.

Different choices for mass and height could change this, maybe up to a factor of 2.

[5 points] Under normal conditions, the blood flow in a certain artery is 26 mL/s. If the diameter of this artery shrinks by 10% (due to plaque formation), what will be the new blood flow in it? Assume blood pressure is unchanged.

mL/s

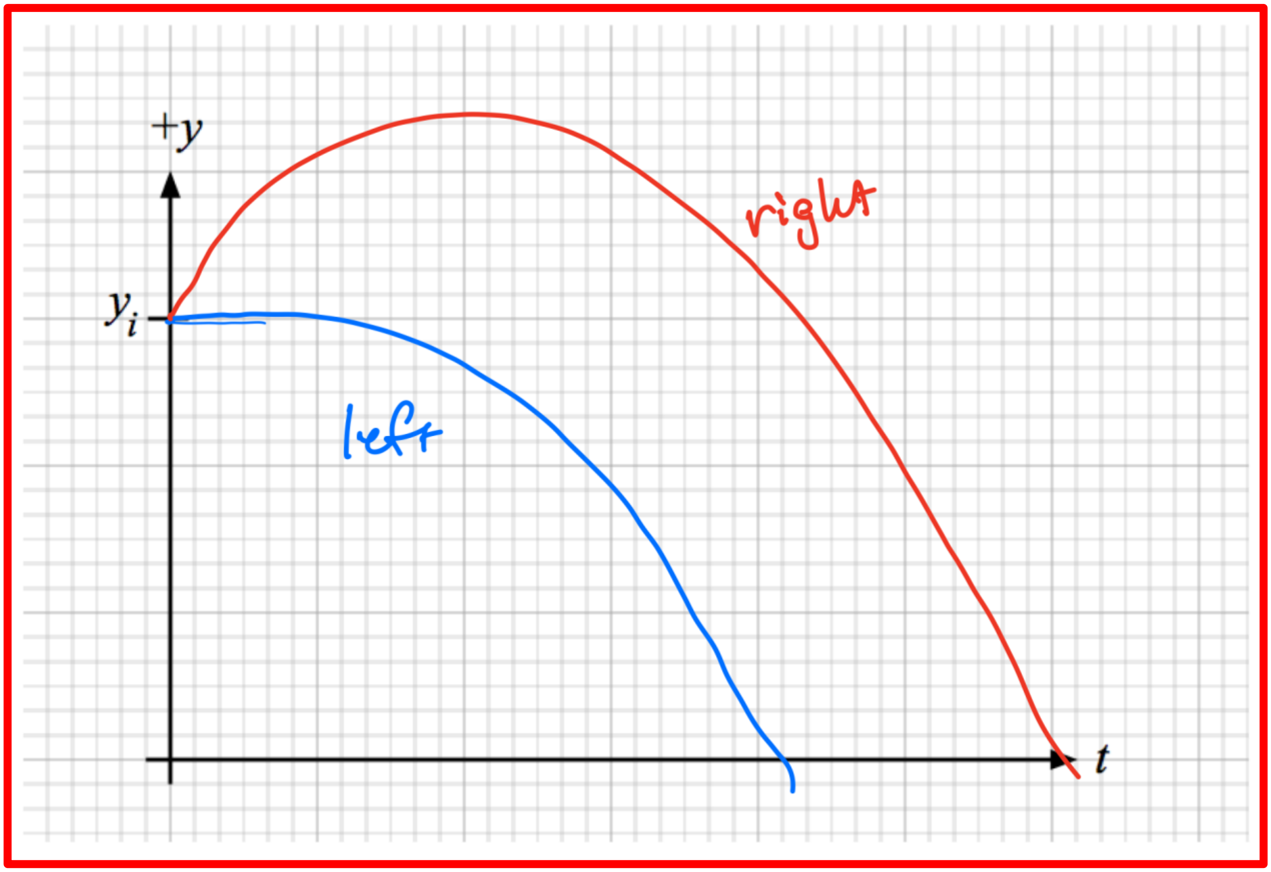
Use the HP equation: $Q = \frac{\pi R^4 \Delta p}{8 \mu L}$.

With fixed pressure, length and viscosity but a smaller radius ($R_B = 0.9 R_A$), this becomes $\frac{Q_B}{Q_A} = \frac{R_B^4}{R_A^4} = \left(\frac{R_B}{R_A}\right)^4 = \left(\frac{0.9 R_A}{R_A}\right)^4 = (0.9)^4 = 0.66$. That is, the new flow is 0.66x the old flow: $Q_B = 0.66 Q_A = 0.66 \left(26 \frac{\text{mL}}{\text{s}}\right) = 17$ mL/s

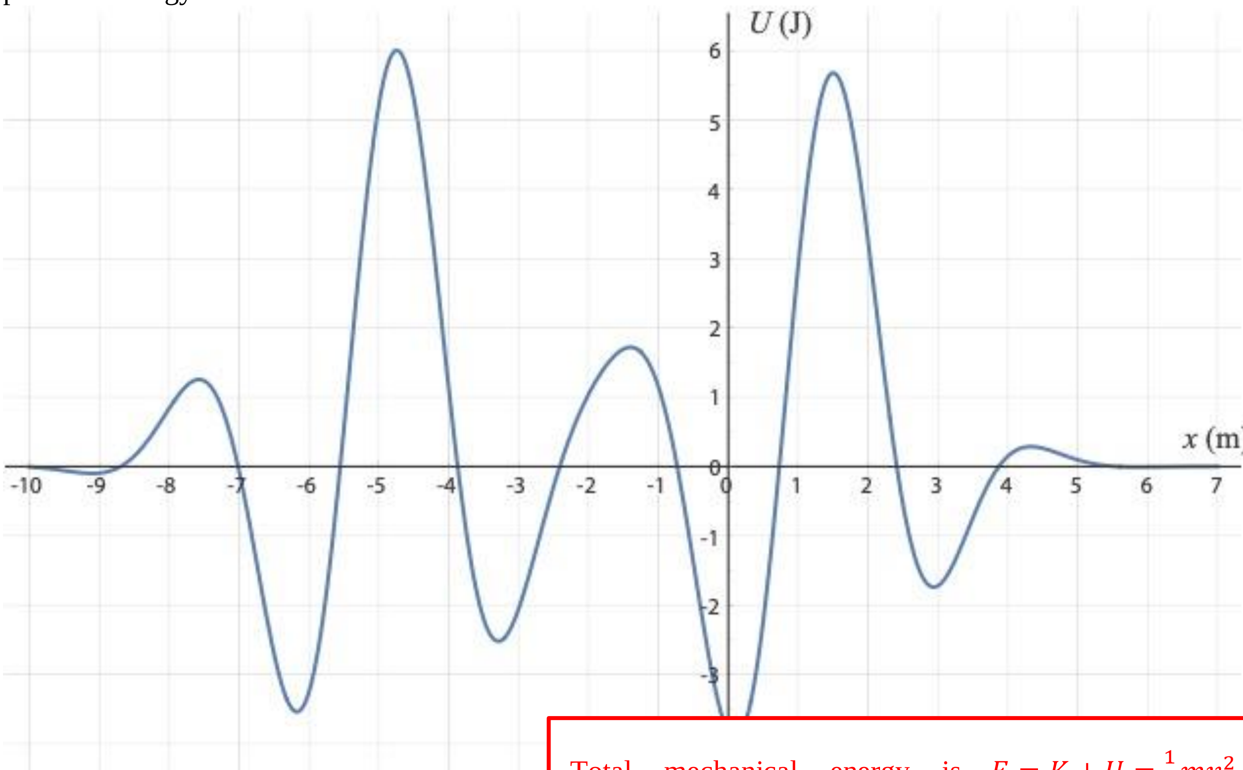
[6 points] You hold two blocks in your left and right hands, each a height y_i above the floor. At $t = 0$, simultaneously, you:

1. let go of the left-hand block, and
2. toss the right-hand block vertically up into the air.

Sketch the position of the blocks as a function of time. Please label which is "left" and which is "right".



[8 points] A 24.0 kg object is released at position $x = -3.0$ m with speed 0.5 m/s, in the presence of the potential energy shown below.



Find (approximately if necessary, since you're

The leftmost turning point (the left edge of the

$x_L =$	m

The rightmost turning point (the right edge of

$x_R =$	m

Total mechanical energy is $E = K + U = \frac{1}{2}mv_i^2 + U(x_i) = \frac{1}{2}(24kg)\left(0.5\frac{m}{s}\right)^2 + (-2.0\text{ J}) = 3J - 2J = 1J.$

Looking at the graph, a line at 1J would intersect $U(x)$ at $x_L = -4.0$ m and $x_R = -2.0$ m.

Max K occurs at min U within this range; that looks like around $x_{\max K} = -3.25$ m or so.

Max force occurs when the slope of $U(x)$ is largest (ignoring sign, since we're only looking at the magnitude); this is at the leftmost edge of the range ($x_{\max F} = -4.0$ m)

The location (within the object's range) where the object has maximum kinetic energy:

$x_{\max K} =$	m

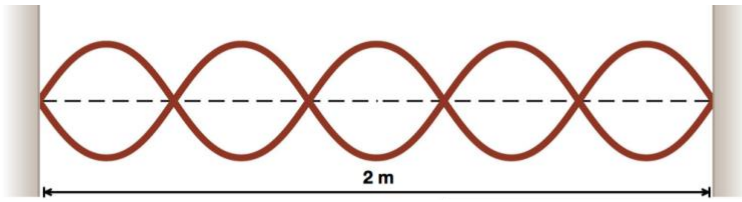
The location (within the object's range) where the object experiences the largest magnitude force:

$x_{\max F} =$	m

Problem 2: Short answer / multiple choice

Limited or no partial credit.

[7 points] A standing wave on a rope looks like this:



If the rope has a mass of 0.1 kg and is stretched to a tension of 500 N, what is the frequency of this standing wave?

Hz

This rope has linear mass density $\mu = \frac{M}{L} = \frac{0.1\,kg}{2m} = 0.05\,kg/m$ so its wave speed is $v = \sqrt{T/\mu} = \sqrt{(500N)/(0.05\frac{kg}{m})} = 100$ m/s.

From the standing wave equation, the fundamental frequency is $f_1 = \frac{v}{2L} = \frac{100\,m/s}{2(2m)} = 25$ Hz.

The pictured standing wave is the 5$^{\text{th}}$ mode (5 antinodes, so $m = 5$) so its frequency is $f_5 = 5f_1 =$ 125 Hz

[6 points] Which of these functions describes a wave that travels in the +z direction? Choose all that are correct. Read carefully!

◯	$\sin(kx)\cos(\omega t)$
X	$\exp(-(kz-\omega t)^2)$
◯	$\sin(ky-\omega t)$
◯	$2\cos(kz)\cos(\omega t)$
◯	$1/(kz+\omega t)^2$

Only (2) is a traveling wave in the +z direction.

(1) and (4) are standing waves (not traveling)

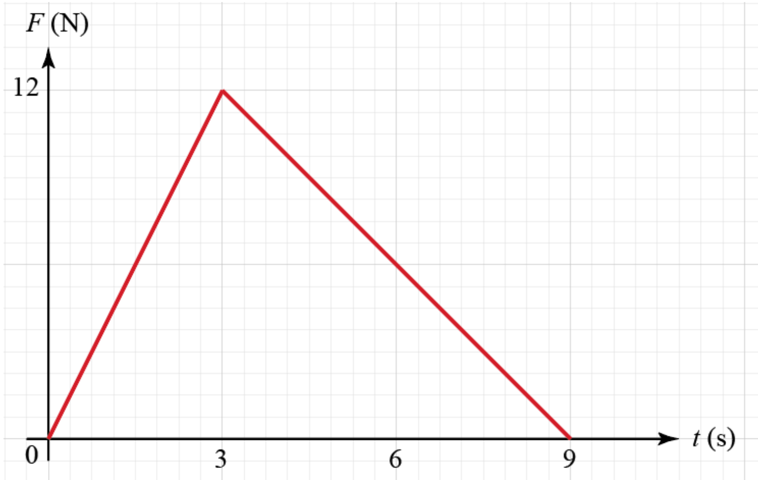
(3) is a traveling wave but in the +y direction.

(5) is a traveling wave but in the -z direction.

Note: Read carefully and contrast the problem on this page and on the following page.

[6 points] A 3 kg object, starting from rest, experiences the force $F(t)$ shown. What is its speed afterwards? Note that the plot shows force vs *time*.

	m/s



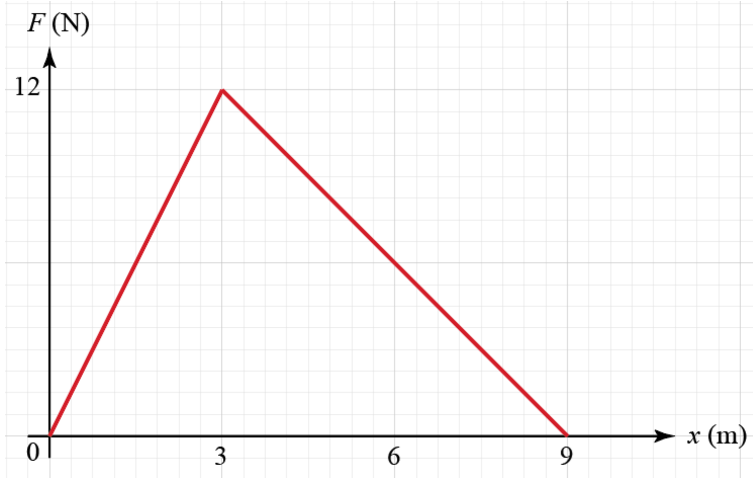
Since it's F(t), use impulse-momentum with $J = \int F\,dt = \frac{1}{2}(9s)(12N) = 54\ Ns.$

Since $v_i = 0$, $v_f = \frac{p_f}{m} = \frac{p_i + J}{m} = \frac{0 + 54\ Ns}{3\ kg} = 18$ m/s

Note: Read carefully and contrast the problem on this page and on the previous page.

[6 points] A 3 kg object, starting from rest, experiences the force $F(x)$ shown. What is its speed afterwards? Note that the plot shows force vs *distance*.

m/s



Since it's F(x), use work-energy with $W = \int F\, dx = \frac{1}{2}(9\text{m})(12\text{N}) = 54\text{ Nm}$.

Since $v_i = 0$, $\frac{1}{2}mv_f^2 = K_f = K_i + W = 0 + W \rightarrow v_f = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2\,(54\text{ Nm})}{(3\ kg)}} = \sqrt{36\,\frac{m^2}{s^2}} = 6$ m/s

Problem 3: Kinematics

Starting from rest at $x = 0$ and $t = 0$,

- In phase 1, you accelerate at a rate of $+2$ m/s^2 for 6 seconds; then
- In phase 2, you accelerate at a rate -6 m/s^2 (that is, 3× as hard and in the opposite direction) for a long time.

A. [6 points] Find your position x_6 and velocity v_6 at the end of phase 1 (that is, at $t = 6$ s).

$x_6 =$	m
$v_6 =$	m/s

Phase 1: $x_1(t) = \frac{1}{2}\left(2\frac{m}{s^2}\right)t^2$ and $v_1(t) = \left(2\frac{m}{s}\right)t$.

So at the end of phase 1 ($t = 6s$), $x_6 = \frac{1}{2}\left(2\frac{m}{s^2}\right)(6s)^2 = 36\ m$ and $v_6 = \left(2\frac{m}{s^2}\right)(6s) = 12\frac{m}{s}$.

B. [9 points] At some point in phase 2, you will turn around and head back towards your starting point. What is the maximum value of x that you reach, and at what time does this occur?

$x_{\max} =$	m
$t_{\max} =$	s

Phase 2 starts from the end of phase 1, so

$x_2(t) = x_6 + v_6(\Delta t) + \frac{1}{2}a_2(\Delta t)^2 = (36m) + \left(12\frac{m}{s}\right)(\Delta t) + \frac{1}{2}\left(-6\frac{m}{s^2}\right)(\Delta t)^2 = 36 + 12\ \Delta t - 3\ \Delta t^2$ (dropping units at the end), where $\Delta t = t - 6s$ is the time elapsed since phase 2 started. The corresponding expression vor velocity is $v_2(t) = v_6 + a_2(\Delta t) = \left(12\frac{m}{s}\right) - \left(6\frac{m}{s^2}\right)\Delta t$.

These can be written more explicitly as $x_2(t) = 36 + 12(t-6) - 3(t-6)^2 = -144m + \left(48\frac{m}{s}\right)t - \left(\frac{3m}{s^2}\right)t^2$ and $v_2(t) = 48\frac{m}{s} - \left(6\frac{m}{s^2}\right)t$, but it's probably easier to deal with the smaller numbers in the Δt version.

Max x_2 occurs when $v_2 = 0$; setting $v_2(\Delta t) = 0$ and solving gives $\Delta t = 2\ s$, or $t = 6s + \Delta t = 8\ s$. At this time, the position is $x_2(2s) = 36 + 12(2) - 3(2^2) = 48$ m

C. [10 points] You will eventually return to your starting position ($x = 0$). Find the time at which that occurs, and your velocity at that time.

$t_{\text{return}} =$	s
$v_{\text{return}} =$	m/s

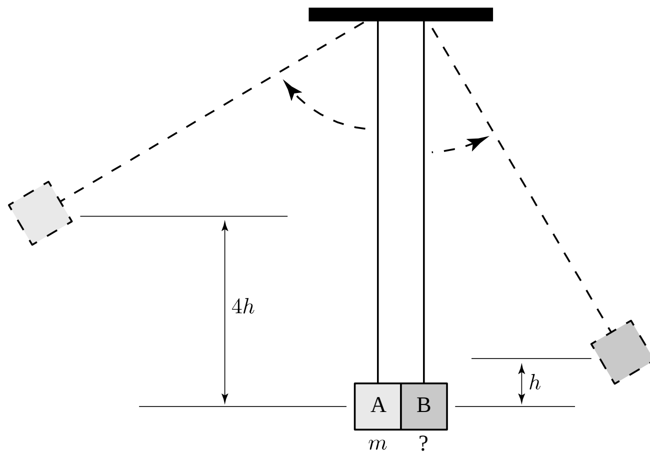
We are trying to find the time at which $x_2(t) = 0 \rightarrow 36 + 12\,\Delta\mathrm{t} - 3\Delta\mathrm{t}^2 = 0$. Divide by –3 to simplify: $\Delta t^2 - 4\Delta t - 12 = 0$.

Solve this by factoring into $(\Delta t - 6)(\Delta t + 2) = 0$ or by using the quadratic formula: $\Delta t = \frac{-(-4)\pm\sqrt{(-4)^2-4(1)(-12)}}{2(1)} = \frac{4\pm\sqrt{16+48}}{2} = \frac{4\pm 8}{2}$. Either method yields two solutions: $\Delta t = -2$ s (nonphysical, since it's *before* phase 2 started) and $\Delta t = 6$ s (the answer we want). The absolute time is $t = 6s + \Delta t = 12\ s$.

At that time, the velocity is $v_2(6s) = 12 - 6(6) = -24$ m/s

Problem 4: Exploding firecracker

Two blocks are hung by strings of length L, and a small firecracker is placed between them. When the firecracker explodes, block A (mass m) swings out and up a distance $4h$; block B (unknown mass) swings out and up a distance h. In the parts below, give your answers symbolically in terms of m and/or h (plus constants, as needed).



A. [10 points] What is the mass of block B?

$m_B =$	

I am writing m_A and h_A for block A's mass and height (to make the symmetrical treatment of blocks A and B clear), but in the problem statement (and in the final solution) these are m and $4h$ respectively.

Conserve momentum (but NOT energy) during the explosion: $m_A v_A = m_B v_B$

Conserve energy for block A after the explosion: $\frac{1}{2} m_A v_A^2 = m_A g h_A \rightarrow v_A = \sqrt{2gh_A}$

Conserve energy for block B similarly: $v_B = \sqrt{2gh_B}$

Combining these three equations: $m_A\left(\sqrt{2gh_A}\right) = m_B\left(\sqrt{2gh_B}\right) \rightarrow m_B = m_A\sqrt{\frac{h_A}{h_B}} = m\sqrt{\frac{4h}{h}} = 2m$

B. [8 points] How much energy Y was released by the firecracker?

$Y =$	

Setting the zero point for gravitational potential energy at the initial positions of the blocks:

$$E_i = K_i + U_i = 0$$

$E_f = K_f + U_f = 0 + m_A g h_A + m_B g h_B = mg(4h) + (2m)gh = 6mgh.$

So: $6mgh$ worth of energy has been added to system; this must have come from the firecracker's explosion: $Y = 6mgh$

C. [7 points] To reduce the carbon footprint of this experiment, you want to replace the *original* firecracker with a small, stiff spring k. You will place the spring between the masses, compress it by a certain amount, and then suddenly release it. Will this work (that is, will this produce the same h's as the original firecracker?), and if so how much should you compress this spring (in terms of *m*, *h* and/or *k* and constants)?

X	Yes, this will mimic the firecracker *provided* you use a compression s =_________
◯	No, there is no way for a spring to produce the same effect as a firecracker.

The only role of the firecracker is to dump an energy $Y = 6mgh$ into the system while pushing the blocks apart. The spring can accomplish the same thing if you initially store the same amount of energy in it (though the energy is stored as elastic potential energy, rather than as the chemical energy of the gunpowder in the firecracker): set $\frac{1}{2}ks^2 = Y = 6mgh \to s = \sqrt{12mgh/k} = 2\sqrt{3mgh/k}$

Extra credit. [5 points] You replace the firecracker with a larger one, which has an explosion that is exactly twice as strong (that is, yielding twice as much energy.) What happens to h_A and h_B, the heights of masses A and B after the explosion, compared to the original scenario?

◯	Both h_A and h_B increase by a factor of 4.
X	Both h_A and h_B double.
◯	Both h_A and h_B increase by a factor of $\sqrt{2}$.
◯	Both h_A and h_B increase, but by different factors.

Looking at the equation at the end of part 1 (in blue), we see that $m_A\sqrt{h_A} = m_B\sqrt{h_B} \to \sqrt{h_A} = 2\sqrt{h_B}$ or $h_B = \frac{1}{4}h_B$. Since they're proportional, they both increase by the same factor.

At the end, all the firecracker energy is transformed into gravitational potential energy, so (since U_g is proportional to h), this will double each of the h's.

If you don't like proportionality arguments you can re-solve with double the yield energy (though it's more work): $K_f - K_i = m_A g h_A + m_B g h_B = mgh_A + (2m)g\left(\frac{1}{4}h_A\right) = \frac{3}{2}mgh_A = 2Y = 12mgh \to h_A = 8h$ (vs $4h$ originally)and $h_B = \frac{1}{4}h_A = 2h$ (vs h originally). This shows explicitly that the new h's are exactly double the original ones.

Problem 5: Lawn Chair Larry

Lawn Chair Larry was a Dallas man who tied helium-filled weather balloons to a lawn chair and floated himself to a height of 16000 ft.*

A. [5 points] At ground level, Larry filled each weather balloon with 1.5 m^3 of helium. Assuming the mass of an *empty* balloon was 20 g, and that each balloon was filled at STP† (where the density of helium is 0.179 kg/m^3), what was the total mass of each *filled* balloon?

$m_1 =$	kg

$m_{He} = \rho V = \left(0.179\,\frac{kg}{m^3}\right)(1.5\,m^3) = 0.268$ kg.

$m_{balloon} = M_{He} + 20g = 0.288$ kg

B. [5 points] Calculate the density of air at Larry's cruising altitude, where $p = 55000$ Pa and $T = -17$ ºC. Treat air as an ideal gas with an effective mass of 29 g/mole.‡

$\rho =$	kg/m^3

$pV = nRT \to \frac{n}{V} = \frac{p}{RT} = \frac{55000\,Pa}{(8.31\,J/mol/K)(256\,K)} = 25.9\,\frac{mol}{m^3}$; convert this to a density by using the molar mass: $\rho = m\frac{n}{V} = \left(29\,\frac{g}{mol}\right)\left(25.9\,\frac{mol}{m^3}\right) = 750\,\frac{g}{m^3} = 0.75\,\frac{kg}{m^3}$.

You can also get this by scaling from the molar volume at STP ($V_{STP} = 22.4$ L): $\rho_{STP} = \left(29\,\frac{g}{mol}\right)\frac{1\,mol}{(22.4\,L)} = 1.30\,\frac{g}{L} = 1.30\,\frac{kg}{m^3}$. From the ideal gas law, density is proportional to pressure and inversely proportional to temperature, so $\frac{\rho_{cruising}}{\rho_{STP}} = \frac{p_{cruising}}{p_{STP}}\frac{T_{STP}}{T_{cruising}} = \frac{(55000)}{(101000)}\frac{(273\,K)}{(256\,K)} = 0.581$: so $\rho_{cruising} = 0.581\,\rho_{STP} = 0.581\,\left(1.30\,\frac{kg}{m^3}\right) = 0.75\,\frac{kg}{m^3}$

* I am not making this up. Larry survived his flight but was fined $1500 by the FAA. This was some time ago, when $1500 was worth something.

† STP is "Standard Temperature and Pressure", which is defined to be a temperature of 0 ºC and a pressure of 1 atm.

‡ For comparison – or in case you want to scale from the value at STP – the density of air at STP is 1.30 kg/m^3.

C. [5 points] Weather balloons expand a limited amount as they rise: each of Larry's balloons doubled in volume from ground level to cruising altitude. How much buoyant force did each balloon provide?

$F_{b1} =$	N

$$F_{b1} = \rho_{cruising} V_{balloon} g = \left(0.75 \frac{kg}{m^3}\right)(3.0 m^3)\left(9.8 \frac{N}{kg}\right) = 22\ N$$

D. [10 points] Larry attached 30 balloons to his chair. Calculate the total combined mass of Larry, his bb gun (used to pop the balloons when he wanted to descend) and his cargo of Miller Lite.

$m =$	kg

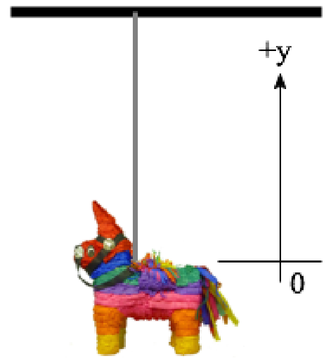
Total buoyant force: $F_b = 30 F_{b1} = 30(22N) = 660$ N

This must lift the helium PLUS Larry and his cargo.

From part A, the helium itself weighs $W_{He} = 30 * (0.288 kg)\left(9.8 \frac{N}{kg}\right) = 85\ N$.

So there is a remaining lift force of 660-85 = 555 N, which corresponds to a mass of 585 N / 9.8 N/kg = 59 kg.

Problem 6: Piñata



A piñata ($m = 2.0$ kg) hangs by an elastic rope ($k = 40$ N/m). Use its equilibrium position as the origin of the y-coordinate system, with +y pointing upward.

A. [6 points] Calculate the angular frequency, frequency, and period for *vertical* oscillation of the piñata.

$\omega =$	rad/s
$f =$	Hz
$T =$	s

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{40 N/m}{2\ kg}} = \sqrt{20\frac{1}{s^2}} = 4.47\,\frac{\text{rad}}{\text{s}};\ f = \frac{\omega}{2\pi} = \frac{4.47 rad/s}{2\pi\, rad} = 0.712\ Hz;$$

$$T = \frac{1}{f} = 1.40\ s$$

B. [8 points] A helpful parent gently lifts the piñata by 10 cm, and then lets go of it (from rest) at $t = 0$. Write down the functions $y(t)$ and $v(t)$ that describe the piñata's position and velocity as a function of time. Use *only numbers* in your answer: no symbols (other than t for time).[§]

$y_B(t) =$	
$v_B(t) =$	

$y(t) = (0.1)\cos(4.47\ t)$ and $v(t) = -(0.45)\sin(4.47\ t)$

You can get this from the canonical forms with $A = 0.1$ m and $\omega = 4.47$ rad/s The factor in from of the sine function in the canonical form is $-A\omega = -(0.1\ m)(4.47\ rad/s) = -0.45$ m/s

[§] If you wish, you *may* use our "standard" form (the one that starts from the form $A\cos(\omega t + \phi_0)$), but this is not required. We will accept *any* correct function. That is, you do *not* need to explicitly derive amplitude and phase to get full credit, provided what you write down is mathematically equivalent to the correct solution.

C. [8 points] With the piñata hanging motionless, a sugar-crazed pre-teen whacks it directly upwards with a baseball bat, instantly giving it an upward speed of 3.0 m/s at $t = 0$. Write down the functions $y(t)$ and $v(t)$ that describe the piñata's position and velocity as a function of time. Use *only numbers* in your answer: no symbols (other than t for time).**

$y_C(t) =$	
$v_C(t) =$	

$y(t) = 0.67\sin(4.47\ t)$ and $v(t) = (3.0)\cos(4.47\ t)$

The easiest way is to just write down v(t) by inspection (you know it's a cosine function with starting value (amplitude) 3.0), and then work backwards to find a y(t) whose derivative correctly gives v(t).

Alternatively, you can get this from the canonical forms with initial conditions $x(0) = A\cos\phi_0 = 0$ and $v(0) = -A\omega\sin\phi_0 = +3.0\ m/s$, or $A\sin\phi_0 = \frac{3.0\ m/s}{-(4.47\ rad/s)} = -0.67$ .Solving those two equations yields $A = 0.67$ and $\phi_0 = -\frac{\pi}{2}$, so another correct solution is

$y(t) = 0.67\cos\left(4.47\ t - \frac{\pi}{2}\right)$ and $v(t) = -3\sin\left(4.47\ t - \frac{\pi}{2}\right)$.

Since $\cos\left(\theta - \frac{\pi}{2}\right) = \sin\theta$ and $\sin\left(\theta - \frac{\pi}{2}\right) = -\cos\theta$, these are equivalent to the function in the first line of the solutions box. But more work to obtain.

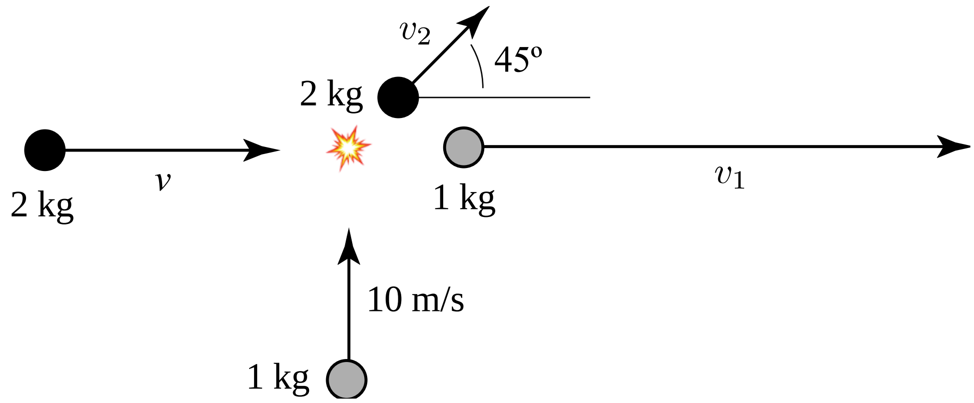
D. [3 points] Candy is gradually leaking out of the piñata as it oscillates. What happens to the frequency of its oscillation?

X	f increases.
◯	f is unchanged.
◯	f decreases.

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. As candy leaks out, m decreases, so f increases.

** See previous footnote. If you can visualize the motion, it may be easier to just guess the proper function and write it down directly rather than doing a lot of math.

Problem 7: Elastic collision



Objects 1 and 2 (with masses 1.0 and 2.0 kg), collide *perfectly elastically* at right angles. The 1 kg object is deflected by 90º; the 2 kg object is deflected by 45º. Object 1 enters with a speed of 10 m/s.

A. [12 points] Use conservation of momentum and energy to write down three equations involving the three unknowns: the initial speed (v) of object 2 and the final speeds (v_1 and v_2) of each object. It's not great notation, but you can drop units to make your equations more readable.

Equation 1:	
Equation 2:	
Equation 3:	

Momentum x: $(2kg)v = (1kg)v_1 + (2\ kg)v_2 \cos 45$.

Momentum y: $(1\ kg)\left(10\frac{m}{s}\right) = (2kg)v_2 \sin 45$.

Energy: $\frac{1}{2}(1\ kg)\left(10\frac{m}{2}\right)^2 + \frac{1}{2}(2\ kg)v^2 = \frac{1}{2}(1kg)v_1^2 + \frac{1}{2}(2\ kg)v_2^2$.

Simplify these by canceling or dropping all the units, factors of two, and writing values for the trig functions:

(1): $2v = v_1 + \frac{2}{\sqrt{2}}v_2$

(2): $10 = \frac{2}{\sqrt{2}}v_2$

(3): $100 + 2v^2 = v_1^2 + 2v_2^2$

B. [13 points] Solve your equations to give values for v, v_1 and v_2.[††]

$v =$	m/s
$v_1 =$	m/s
$v_2 =$	m/s

1. Solve the y-momentum equation (2) to give $v_2 = \frac{10}{\sqrt{2}} = 7.1$

2. Plug this into eqns (1) and (3) to give two new equations (in v1 and v only):

(1'): $2v = v_1 + \frac{2}{\sqrt{2}} v_2 = v_1 + \frac{2}{\sqrt{2}}\left(\frac{10}{\sqrt{2}}\right) = v_1 + 10$

(3'): $100 + 2v^2 = v_1^2 + 2v_2^2 = v_1^2 + 2\left(\frac{10}{\sqrt{2}}\right)^2 = v_1^2 + 100.$

3. Cancel 100 from both sides of (3') to give $2v^2 = v_1^2 \to v_1 = \sqrt{2}v$

4. Plug this into (1') to give an equation that contains only v:

(1''): $2v = v_1 + 10 = \left(\sqrt{2}v\right) + 10 \to v = \frac{10}{2-\sqrt{2}} = 17.1$

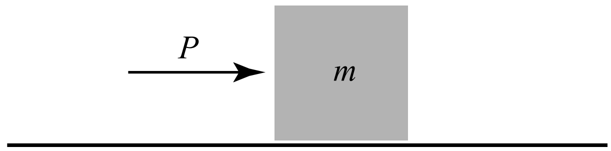
5. Insert this value in the expression from step 3:

$v_1 = \sqrt{2}v = \sqrt{2}(17.1) = 24.1$

[††] It's a good idea to substitute your solved values back into the original equations in part A to make sure you haven't made an algebraic error somewhere.

Problem 8: Pushing on a block

Scenario A: A block m is placed on a horizontal surface. You push on it with small force $P = \frac{1}{20}mg$. It doesn't move.



A1. [6 points] Calculate the normal force N and the static frictional force F_s on the block from the surface below. Give your answers as a multiple of mg.

$N =$	mg
$F_s =$	mg

Use a normal (untilted) coordinate system.

In the vertical direction: $N - mg = ma_y = 0 \rightarrow N = mg$

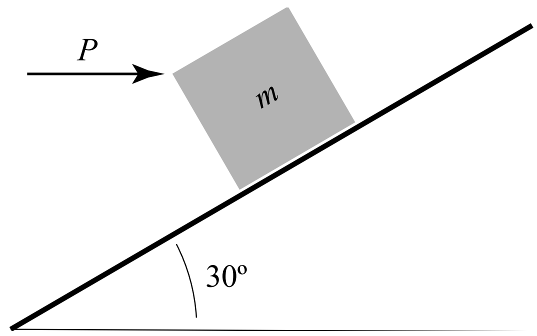
In the horizonal direction: $P - F_s = ma_x = 0 \rightarrow F_s = P = \frac{1}{20}mg$

A2. [6 points] What can you say about μ_s, the static coefficient of friction between the block and the surface?

○	You can't draw any conclusions about μ_s.
○	$\mu_s \leq$ ______
○	$\mu_s =$ ______
X	$\mu_s \geq$ ______

We know that $F_s \leq F_{smax} = \mu_s N \rightarrow \frac{1}{20}mg \leq \mu_s(mg) \rightarrow \mu_s \geq \frac{1}{20} = 0.05$

Scenario B: A block m is placed on a different sloped surface making an angle $\theta = 30°$ to the horizontal. You push on it with a small *horizontal* force $P = \frac{1}{20} mg$. It doesn't move.



B1. [6 points] Calculate the normal force N and the static frictional force F_s on the block from the surface below. Give your answers as a multiple of mg.‡‡

$N =$	mg
$F_s =$	mg

Repeat A1, except use a tilted coordinate system (not mandatory, but *much* easier), so you have to split P and weight W into components:

Perpendicular to the surface: $N - P\sin\theta - W\cos\theta = ma_\perp = 0 \rightarrow N = W\cos\theta + P\sin\theta = mg\cos 30 + \left(\frac{1}{20}mg\right)\sin 30 = \left(\frac{\sqrt{3}}{2} + \frac{1}{40}\right)mg = 0.89\ mg$.

Parallel to the surface (assuming Fs points UP the surface, because the pushing force is pretty small): $P\cos\theta - W\sin\theta - F_s = ma_\parallel = 0 \rightarrow F_s = W\sin\theta - P\cos\theta = mg\sin 30 - \left(\frac{1}{20}mg\right)\cos 30 = \left(\frac{1}{2} - \frac{\sqrt{3}}{40}\right)mg = 0.46\ mg$. Since Fs>0, it was correct to assume Fs points UP the surface.

‡‡ Hint: your algebra will be significantly easier if you use a tilted coordinate system that's aligned with the slope.

B2. [7 points] What can you say about μ_s, the static coefficient of friction between the block and the surface?

◯	You can't draw any conclusions about μ_s.
◯	$\mu_s \leq$ ______
◯	$\mu_s =$ ______
X	$\mu_s \geq$ ______

Like A2: from $F_s \leq F_{smax} = \mu_s N \rightarrow 0.46\,mg \leq \mu_s(0.89\,mg) \rightarrow \mu_s \geq \frac{0.46}{0.89} = 0.51$

Extra credit. [5 points] With such a small pushing force, you definitely need friction to prevent the block from slipping. However, if you push on the block hard enough, the block will remain stationary even *without* friction. What is the critical pushing force that yields $F_s = 0$?

Critical $P =$	mg

Look at the equation in blue in B1: $F_s = W\sin\theta - P\cos\theta = 0 \rightarrow P = W\tan\theta = \tan 30\ mg = 0.58\,mg$