Math 2551 Final Review Fall 2023

Math 2550 students should work problems 1-18 only.

1. Find $\mathbf{T}, \mathbf{N}$, and the curvature κ for $\mathbf{r}(t) = 3\sin(t)\mathbf{i} + 3\cos(t)\mathbf{j} + 4t\mathbf{k}$.

2. Find the limit: $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2 - y^2 + 5}{x^2 + y^2 + 2}$

3. **True** or **False**: Suppose $\displaystyle\lim_{(x,y)\to(a,b)} f(x,y) = 5$ along the lines $x = a$ and $y = b$ and $f(a,b) = 5$. Then f must be continuous at (a,b).

4. Find all second-order partial derivatives for $w = x\sin(x^2 y)$

5. Evaluate $\dfrac{dw}{dt}$ for $w = 2ye^x - \ln(z)$ if $x(t) = \ln(t^2+1), y = \arctan(t), z = e^t$ at $t = 1$.

6. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for the surface $z^3 - xy + yz + y^3 - 2 = 0$ at the point $(1,1,1)$.

7. Based on the contour plot below for a function $f(x,y)$, determine the sign $(+, -, 0)$ of

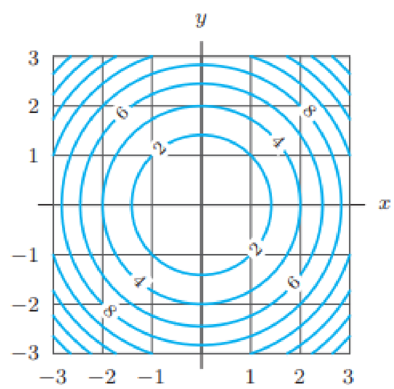
   (a) $f_x(0,2)$

   (b) $f_y(2,-2)$

   (c) the rate of change of f at $(-2,0)$ in the direction towards $(-3,3)$

   (d) $Df_{\langle 1/\sqrt{2}, 1/\sqrt{2}\rangle}(-1,-1)$

   (e) the rate of change of f in the direction tangent to the level curve through $(0,1)$



8. Find the derivative of the function $f(x,y) = 2xy - 3y^2$ in the direction $4\mathbf{i} + 3\mathbf{j}$ at the point $(5,5)$.

9. Find the tangent plane and normal line at $(2,0,2)$ of the surface $2z - x^2 = 0$.

10. Find all local maxima, local minima, and saddle points for $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.

11. Find the absolute maximum and minimum of $f(x,y) = 2x^2 - 4x + y^2 - 4y + 1$ on the closed triangular region bounded by the lines $x = 0, y = 2$, and $y = 2x$.

12. Find the maximum value of $f(x,y) = 49 - x^2 - y^2$ on the line $x + 3y = 10$.

13. Write an iterated integral using (a) vertical cross-sections and (b) horizontal cross-sections whose region of integration is the region bounded by $y = \sqrt{x}, y = 0, x = 9$.

14. Sketch the region of integration and evaluate the integral $\displaystyle\int_0^1 \int_0^{y^2} 3y^3 e^{xy}\,dx\,dy$.

15. Change to polar coordinates and evaluate the integral $\displaystyle\int_{-1}^1 \int_0^{\sqrt{1-x^2}} dy\,dx$.

16. Integrate the function $f(x,y,z) = 3 - 4x$ over the region below $z = 4 - xy$ and above the rectangle $0 \le x \le 2, 0 \le y \le 1$ in the xy-plane.

17. Find the volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 2$ and outside the cylinder $x^2 + y^2 = 1$.

18. Use the change of coordinates $x(u, v) = \frac{u}{v}, y(u, v) = uv$ to evaluate the integral $\iint_R \left(\sqrt{\frac{y}{x}} + \sqrt{xy}\right) dA$, where R is the region in the first quadrant bounded by $xy = 1, xy = 9, y = x$, and $y = 4x$.

19. Evaluate the line integral $\int_C (xy + y + z)\, ds$ along the curve $\mathbf{r}(t) = 2t\mathbf{i} + t\mathbf{j} + (2 - 2t)\mathbf{k}$ for $0 \le t \le 1$.

20. Find the flow of the field $\mathbf{F} = \langle -4xy, 8y, 2\rangle$ along the curve $\mathbf{r}(t) = \langle t, t^2, 1\rangle, 0 \le t \le 2$.

21. Find the counterclockwise circulation and the outward flux of the field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ around/through the unit circle centered at the origin.

22. Find the potential function for $\mathbf{F} = e^{y+2z}(\mathbf{i} + x\mathbf{j} + 2x\mathbf{k})$.

23. Use Green's Theorem to find counterclockwise circulation and outward flux of the field $\mathbf{F} = \langle y^2 - x^2, x^2 + y^2\rangle$ for the curve C enclosing the region bounded by $y = 0, x = 3$, and $y = x$.

24. Let f be a function of three variables and let $\mathbf{F}$ and $\mathbf{G}$ be vector fields in $\mathbb{R}^3$. Which of the following expressions make mathematical sense? If you can compute any of them, do so.

    (a) curl(div($\mathbf{F}$))
    (b) curl(∇f)
    (c) div($\mathbf{F} \cdot \mathbf{G}$)
    (d) curl(div(f))
    (e) div(curl($\mathbf{G}$))
    (f) div(∇f)

25. Use a parameterization to write a double integral for the area of the surface S which is the portion of the cone $z = 2\sqrt{x^2 + y^2}$ between the planes $z = 2$ and $z = 6$.

26. Evaluate $\iint_S 2y\, d\sigma$ over the surface S which is the part of the cylinder $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.

27. Let S be the surface that consists of the part of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane and below the cone $z = 3\sqrt{x^2 + y^2}$.

    (a) Sketch S.
    (b) Find a parameterization of S.
    (c) Calculate the area of S.

28. Let S be the surface consisting of the top half $z \ge 0$ of the sphere $x^2 + y^2 + z^2 = 9$, together with the disk $x^2 + y^2 \le 9, z = 0$, its base in the xy-plane. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$, where $\mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + 3x^2y\mathbf{j} + z^3\mathbf{k}$.

29. Let S be the part of the surface $z = 4x^2 + y^2 - 4$ beneath the plane $z = 5$. Let C be the bounding curve of S in the plane $z = 5$, traversed counterclockwise and suppose S is oriented accordingly (normals towards the z-axis). Let $\mathbf{F}(x, y, z) = \langle 2y, 4x, e^x\rangle$. Use Stokes' Theorem to evaluate the curl integral

$$\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, d\sigma.$$

30. Let S be the surface of the cylinder defined by $y^2 + z^2 = 4$ between the planes $x = -1$ and $x = 3$. Let $\mathbf{F}(x, y, z) = e^{xy}\mathbf{i} + e^x y\mathbf{j} + e^x z\mathbf{k}$.

    (a) Sketch S.
    (b) Find a parameterization of S.
    (c) Let $\mathbf{n}$ be an outward pointing unit normal for S. Evaluate

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, d\sigma$$

    by direct calculation (do not use the Divergence Theorem).