Math 1552, Integral Calculus

Practice Problems for Midterm #2

Sections 6.1-6.2, 8.2-8.4

NOTE FROM THE INSTRUCTORS Please note that students are expected to also understand how to integrate with u-substitutions, as this technique may be needed in order to evaluate integrals from the above listed sections.

1. Content Recap

(a) The general formulas for finding the volume of a solid of revolution using the disk method are given by:

(b) The general formulas for finding the volume of a solid of revolution using the shell method are given by:

(c) In the disk method, the variable of integration ____________ the axis of rotation.

(d) In the shell method, the variable of integration is ____________ of the axis of rotation.

(e) Evaluate an integral using *integration by parts* if:

To choose the value of u, use the rule: ____________.

(f) To evaluate integrals with powers or products of trig functions, use the following trig identities to try to obtain a u-substitution:

(g) If we would evaluate an integral using *trig substitution*, the integral should contain an expression of one of these forms: _____________, _____________, or _____________.

Write out the trig substitution you would use for each form listed above.

2. For each problem below, find the volume of the solid generated by revolving the region R about the given line.

(a) R is the region bounded by the curves $y = x^3$, $x + y = 10$, and the line $y = 1$; about the x-axis

(b) R is the region bounded by the curve $y = x^{2/3} + 1$, the y-axis, and the line $y = 5$; about the y-axis

(c) R is the region bounded by the curves $y^2 = 4x$ and $y = x$; about the x-axis

(d) R is the region bounded by the curves $y = \sqrt{1 - x^2}$ and $x + y = 1$; about the x-axis

(e) R is the region bounded by the curves $y^2 = 4x$ and $y = x$; about the line $x = 4$

(f) R is the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$; about the line $x = -2$

(g) R is the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$; about the line $y = 2$

(h) R is the region bounded by the curves $y = \sqrt{3}x$, $y = \sqrt{4 - x^2}$, and $y = 0$; about the y-axis

3. Evaluate the following integrals using any method we have learned.

(a) $\int \frac{x+2}{x+1} dx$

(b) $\int \sqrt{25 - x^2} dx$

(c) $\int \tan^3(x) \sec^4(x) dx$

(d) $\int x \tan^{-1}(x) dx$

(e) $\int \sin^2(x)\cos^2(x)dx$

(f) $\int (x^2+1)e^{2x}dx$

(g) $\int \frac{dx}{x\sqrt{1+x^2}}$

(h) $\int \sin^3(x)\cos^3(x)\ dx$

(i) $\int x\sin(x)\cos(x)\ dx$

(j) $\int \sec^4(x)\ dx$

(k) $\int \frac{8\ dx}{x^2\sqrt{4-x^2}}$

(l) $\int \frac{8\ dx}{(4x^2+1)^2}$

(m) $\int x^5\cos(x^3)\ dx$

(n) $\int_0^1 \ln(1+x^2)\ dx$

(o) $\int (2x+3)4^{-x}\ dx$

(p) $\int \frac{\sin^3(x)}{\cos^7(x)}\ dx$

(q) $\int \cot^3(x)\csc^3(x)\ dx$

(r) $\int \frac{dx}{e^x\sqrt{e^{2x}-9}}$

Answers

2. (a) $\frac{3790}{21}\pi$ (b) 64π (c) $\frac{32}{3}\pi$

(d) $\frac{\pi}{3}$ (e) $\frac{64}{5}\pi$ (f) $\frac{49}{30}\pi$

(g) $\frac{31}{30}\pi$ (h) $\frac{8\pi\sqrt{3}}{3}$

3. (a) $x + \ln|x+1| + C$

(b) $\frac{25}{2}\sin^{-1}\left(\frac{x}{5}\right) + \frac{x\sqrt{25-x^2}}{2} + C$

(c) $\frac{1}{4}\tan^4(x) + \frac{1}{6}\tan^6(x) + C$

(d) $\frac{x^2}{2}\tan^{-1}(x) - \frac{x}{2} + \frac{1}{2}\tan^{-1}(x) + C$

(e) $\frac{x}{8} - \frac{1}{32}\sin(4x) + C$

(f) $\frac{1}{2}(x^2+1)e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$

(g) $-\ln\left|\frac{\sqrt{1+x^2}}{x} + \frac{1}{x}\right| + C$

(h) $\frac{1}{4}\sin^4(x) - \frac{1}{6}\sin^6(x) + C$

(i) $\frac{x}{2}\sin^2 x - \frac{1}{4}x + \frac{1}{8}\sin 2x + C$

(j) $\tan(x) + \frac{\tan^3(x)}{3} + C$

(k) $\frac{-2\sqrt{4-x^2}}{x} + C$

(l) $2\tan^{-1}(2x) + \frac{4x}{4x^2+1} + C$

(m) $\frac{1}{3}x^3\sin(x^3) + \frac{1}{3}\cos(x^3) + C$

(n) $\ln 2 + \frac{\pi}{2} - 2$

(o) $-4^{-x}\left[\frac{2x+3}{\ln 4} + \frac{2}{(\ln 4)^2}\right] + C$

(p) $\frac{1}{4}\tan^4(x) + \frac{1}{6}\tan^6(x) + C$

(q) $-\frac{1}{5}\csc^5(x) + \frac{1}{3}\csc^3(x) + C$

(r) $\frac{\sqrt{e^{2x}-9}}{9e^x} + C$