MATH 2551 Reading Day Study Session - Fall 2017

1. Let S be the surface that consists of that part of the paraboloid $z = 4 - x^2 - y^2$ above the xy-plane, after we cut out the cone $z = 3\sqrt{x^2 + y^2}$. We emphasize this is the part remaining after you cut out the cone, and you don't include the cone's area.

   (a) Sketch S.

   (b) Find a parametrization for the surface.

   (c) Calculate the area of the surface S.

2. Let S be the surface consisting of the top half ($z \geq 0$) of the sphere $x^2 + y^2 + z^2 = 9$, together with its base in the xy-plane, namely the disc $x^2 + y^2 \leq 9$, $z = 0$. Use the divergence theorem to evaluate
$$\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$$
   where
$$\mathbf{F}(x, y, z) = 3xy^2\mathbf{i} + 3x^2y\mathbf{j} + z^3\mathbf{k}.$$

3. Let S be that part of the surface $z = 4x^2 + y^2 - 4$ beneath the plane $z = 5$. Let C be the bounding curve of S in the plane $z = 5$, traversed counterclockwise. Assume that S is oriented accordingly. Let $\mathbf{F}(x, y, z) = 2y\mathbf{i} = 4x\mathbf{j} + e^x\mathbf{k}$. Use Strokes' Theorem to evaluate the curl integral
$$\int\int_S \nabla \times \mathbf{F} \cdot \mathbf{n} d\sigma.$$

4. Let S be the surface of the cylinder defined by $y^2 + z^2 = 4$ between the planes $x = -1$ and $x = 3$. Let $\mathbf{F}(x, y, z) = e^{xy}\mathbf{i} + e^x y\mathbf{j} + e^x z\mathbf{k}$.

   (a) Sketch S.

   (b) Find a parametrization for S.

   (c) Let $\mathbf{n}$ be an outward pointing normal from S. Evaluate
$$\int\int_S \mathbf{F} \cdot \mathbf{n} d\sigma$$
   by calculation (do not try to use the divergency theorem).

MATH 2551 Reading Day Study Session - Fall 2017 (Solutions)

1. (a) - (b) $\mathbf{r}(\theta, r) = (r\cos\theta, r\sin\theta, 4 - r^2)$; $0 \leq \theta \leq 2\pi$, $1 \leq r \leq 2$. (c) $\frac{\pi}{6}[17^{3/2} - 5^{3/2}]$.

2. $\frac{1458}{5}\pi$

3. -27π

4. (a) - (b) $\mathbf{r}(\theta, x) = (x, 2\cos\theta, 2\sin\theta)$, $0 \leq \theta \leq 2\pi$, $-1 \leq x \leq 3$. (c) $8\pi(e^3 - e^{-1})$.