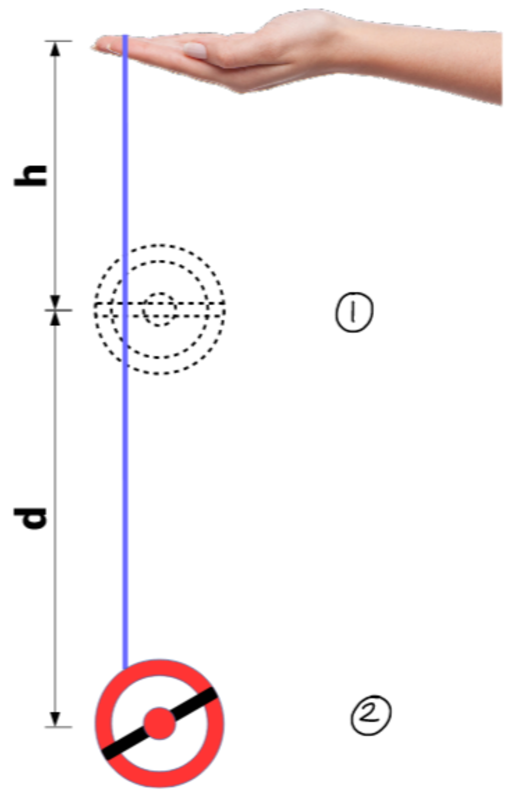
PHYS 2211 Classical Reading Day – Rotational Kinetic Energy

Problem 3 [10 pts] Grader & Score: ____________________

A physics student is playing with a yo-yo. The yo-yo is initially held motionless in mid-air when the student releases the yo-yo and pulls up on the string with a constant force $\vec{F_0}$. Their hand moves up a distance h as the yo-yo falls a distance d under the force of gravity. The yo-yo has mass m, and the mass of the string can be ignored.



A. [10 pts] What is the final translational kinetic energy of the yo-yo after it has fallen a distance d?

B. [10 pts] What is the final rotational kinetic energy of the yo-yo after it has fallen a distance d?

C. [5 pts] In a different experiment, instead of letting the yo-yo drop, a physics student pulls up on the string with a constant force $\vec{F_1}$. Their hand moves up a distance h but the center of mass for the yo-yo doesn't move. Calculate the magnitude of $\vec{F_1}$

Problem 4 [10 pts] Grader & Score: ______________________

This problem is about two spaceships that meet in space, and their angular momentum:

Spaceship "A" is located at the origin. It is a uniform sphere of mass 100 kg and radius 12 m. It spins with angular speed $\omega = 15$ rad/s, and the rotational angular momentum points in the direction given by the unit vector $\hat{l} = \langle 0, 1, 0 \rangle$. The center of A is not moving.

Spaceship "B" is located at position $\langle 50, 300, -200 \rangle$ m. It is a cylinder of mass 50 kg that is not rotating. The center of B is moving with velocity $\langle 30, -20, 100 \rangle$ m/s.

A. [5 pts] Where is the center of mass of the two spaceship system?

B. [5 pts] What is the rotational angular momentum of spaceship A?

C. [10 pts] What is the translational angular momentum of spaceship B about the origin?

D. [5 pts] What is the total angular momentum, about the origin, for the two spaceship system.

Problem 5

A. [8 pts] A solid sphere of steel and a smaller solid sphere with the same mass but (1/4) the radius roll down an incline plane. Both spheres start from rest and at the same time from the same height. Which one will arrive at the bottom first? Please show how you determined this.

Answers

Problem 3

A. [10 pts] What is the final translational kinetic energy of the yo-yo after it has fallen a distance d?

System: disk (point particle) Surroundings: Earth, hand

Initial: ① $K_{trans,i} = 0$

Final: ② $K_{trans,f} = ?$

-1 clerical or included units
-2 minor
-4 major
-8 BTN

Work calculations:

Gravity does work over $\Delta\vec{r} = -d\,\hat{y}$ $W_g = (-mg\hat{y})\cdot(-d\hat{y}) = mgd$

Hand does work over $\Delta\vec{r} = -d\,\hat{y}$ $W_h = (F_o\hat{y})\cdot(-d\hat{y}) = -F_o d$

Energy principle $\Delta E = \Delta K_{trans} = W_g + W_h$

$K_{trans,f} = mgd - F_o d = (mg - F_o)d$

$\boxed{K_{trans,f} = (mg - F)d}$

B. [10 pts] What is the final rotational kinetic energy of the yo-yo after it has fallen a distance d?

System: yo-yo (extended) Surroundings: Earth, hand

Initial: ① $K_{trans,i} = 0$ $K_{rot,i} = 0$

Final: ② $K_{trans,f} = (mg - F)d$ from (a) $K_{rot,f} = ?$

Work calculations

Gravity does work over $\Delta\vec{r} = -d\,\hat{y}$ $W_g = (-mg\hat{y})\cdot(-d\hat{y}) = mgd$

Hand does work over $\Delta\vec{r} = h\hat{y}$ $W_h = (F_o\hat{y})\cdot(h\hat{y}) = F_o h$

Energy principle $\Delta E = \Delta K_{trans} + \Delta K_{rot} = W_g + W_h$

$$\begin{aligned} K_{rot,f} &= W_g + W_h - K_{trans,f} \\ &= mgd + F_o h - (mg - F_o)d \\ &= mgd + F_o h - mgd + F_o d \\ &= F_o(h+d) \end{aligned}$$

$\boxed{K_{rot,f} = F_o(h+d)}$

-1 clerical or included units

C. [5 pts] In a different experiment, instead of letting the yo-yo drop, a physics student pulls up on the string with a constant force $\vec{F}_1$. Their hand moves up a distance h but the center of mass for the yo-yo doesn't move. Calculate the magnitude of $\vec{F}_1$

If the center of mass doesn't move, that means $K_{trans,f} = 0$

Consider the point particle system

$$\Delta K_{trans} = K_{trans,f} = (mg - F_1)d$$

For $K_{trans,f}$ to be zero,

$$|\vec{F}_1| = mg$$

$\boxed{|F_1| = mg}$

Problem 4

This problem is about two spaceships that meet in space, and their angular momentum:

Spaceship "A" is located at the origin. It is a uniform sphere of mass 100 kg and radius 12 m. It spins with angular speed $\omega = 15$ rad/s, and the rotational angular momentum points in the direction given by the unit vector $\hat{l} = \langle 0, 1, 0 \rangle$. The center of A is not moving.

Spaceship "B" is located at position $\langle 50, 300, -200 \rangle$ m. It is a cylinder of mass 50 kg that is not rotating. The center of B is moving with velocity $\langle 30, -20, 100 \rangle$ m/s.

All A. [5 pts] Where is the center of mass of the two spaceship system?

$$\vec{r}_{cm} = \frac{m_A \vec{r}_A + m_B \vec{r}_B}{m_A + m_B}$$

$$= \frac{(100\text{ kg})\langle 0,0,0\rangle\text{m} + (50\text{ kg})\langle 50, 300, -200\rangle\text{m}}{100\text{ kg} + 50\text{ kg}}$$

$$= (16.67, 100, -66.67)\text{ m}$$

$$\boxed{\vec{r}_{cm} = (16.67, 100, -66.67)\text{ m}}$$ −1 missing or incorrect units

B. [5 pts] What is the rotational angular momentum of spaceship A?

$$\vec{L}_{rot,A} = I\vec{\omega}$$

$$= I_{sphere}\, \omega\, \hat{l}$$

$$= \frac{2}{5} m_A R_A^2 \omega\, \hat{l}$$

$$= \frac{2}{5}(100\text{ kg})(12\text{ m})^2\left(15\,\tfrac{\text{rad}}{\text{s}}\right)\langle 0, 1, 0\rangle$$

$$= \langle 0, 86400, 0\rangle\ \text{kg}\cdot\text{m}^2/\text{s}$$

$$\boxed{\vec{L}_{rot,A} = \langle 0, 86400, 0\rangle\ \text{kg}\cdot\text{m}^2/\text{s}}$$ −1 missing or incorrect units

C. [10 pts] What is the translational angular momentum of spaceship B about the origin?

$$\vec{L}_{trans,B} = \vec{r}_B \times \vec{p}_B = m_B(\vec{r}_B \times \vec{v}_B)$$

$$= m_B \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ v_x & v_y & v_z \end{vmatrix} = (50\,kg) \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ 50m & 300m & -200m \\ 30m/s & -20m/s & 100m/s \end{vmatrix}$$

$$= m_B\left[(r_y v_z - r_z v_y)\hat{x} - (r_x v_z - r_z v_x)\hat{y} + (r_x v_y - r_y v_x)\hat{z}\right]$$

$$= (50\,kg)\Big[\big((300m)(100m/s) - (-200m)(-20m/s)\big)\hat{x}$$
$$- \big((50m)(100m/s) - (-200m)(30m/s)\big)\hat{y}$$
$$+ \big((50m)(-20m/s) - (300m)(30m/s)\big)\hat{z}\Big]$$

$$= (1.3\times10^6, -5.5\times10^5, -5.0\times10^5)\ kg\cdot m^2/s$$

$$\boxed{\vec{L}_{trans,B} = (1.3\times10^6, -5.5\times10^5, -5.0\times10^5)\ kg\cdot m^2/s}$$

-1 clerical or missing/incorrect unit
-2 minor

D. [5 pts] What is the total angular momentum, about the origin, for the two spaceship system.

For A: $\vec{L}_{rot,A}$ from (b)
$\vec{L}_{trans,A} = \vec{0}$ because $\vec{r}_A = \vec{0}$

For B: $\vec{L}_{rot,B} = \vec{0}$ because not rotating ($\omega = 0$)
$\vec{L}_{trans,B}$ from (c)

$$\vec{L}_{total} = \vec{L}_{rot,A} + \cancelto{0}{\vec{L}_{trans,A}} + \cancelto{0}{\vec{L}_{rot,B}} + \vec{L}_{trans,B}$$

$$= \langle 0, 8.64\times10^4, 0\rangle\ kg\cdot m^2/s + \langle 1.3\times10^6, -5.5\times10^5, -5.0\times10^5\rangle\ kg\cdot m^2/s$$

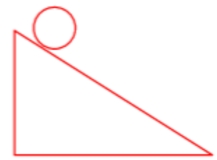
$$= \langle 1.3\times10^6, -4.64\times10^5, -5.0\times10^5\rangle\ kg\cdot m^2/s$$

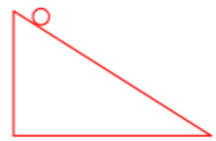
$$\boxed{\vec{L}_{total} = \langle 1.3\times10^6, -4.64\times10^5, -5.0\times10^5\rangle\ kg\cdot m^2/s}$$

-1 missing or incorrect units
* Watch for POE from (b) & (c)

Problem 5

A. [8 pts] A solid sphere of steel and a smaller solid sphere with the same mass but (1/4) the radius roll down an incline plane. Both spheres start from rest and at the same time from the same height. Which one will arrive at the bottom first? Please show how you determined this.





Energy conservation
System: sphere and Earth

$$\Delta E = 0$$

$$\Delta K_{trans} + \Delta K_{rot} + \Delta U_{grav} = 0$$

$$\tfrac{1}{2} m |\vec{v}_f|^2 + \tfrac{1}{2} I |\vec{\omega}_f|^2 - mg\,\Delta h = 0$$

$$|\omega_f| = \frac{2|\vec{v}_f|}{R}$$

$$\tfrac{1}{2} m |\vec{v}_f|^2 + \frac{2I\,|\vec{v}_f|^2}{R^2} - mg\,\Delta h = 0$$

$$\left(\tfrac{1}{2} m + \frac{2I}{R^2}\right)|\vec{v}_f|^2 = mg\,\Delta h$$

$$|\vec{v}_f|^2 = mg\,\Delta h \,\frac{1}{\tfrac{1}{2}m + \frac{2I}{R^2}}$$

sphere
$$I = \tfrac{2}{5} m R^2$$

$$|\vec{v}_f|^2 = mg\,\Delta h \,\frac{1}{\tfrac{1}{2}m + \tfrac{4}{5}m}$$

Doesn't depend on R

They both reach the bottom at the same time

-1 Clerical
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-6 BTN

ALTERNATIVELY
Full points for simply stating that they both reach the bottom at the same time. No work necessary.