A vinyl record has mass $M$ and radius $R$ and rotates about the center of the record with a constant angular velocity $\omega = -\omega_0 \hat{z}$ as indicated in the diagram. You notice a confused yellow jacket with mass $m$ is sitting on the record at the origin. At a time $t = 0$, the yellow jacket starts walking outward from the center of the record. From the perspective of the yellow jacket, she follows a straight line as indicated by the dashed yellow line in the diagram.

A. [5 pts] Calculate the total vector angular momentum of the record about a point $A$ located at the origin at time $t = 0$.

B. [5 pts] Calculate the total vector angular momentum of the yellow jacket about a point $A$ located at the origin at time $t = 0$ (i.e. when the yellow jacket is at the center of the record). The yellow jacket is so small it can be considered a point mass.

C. [10 pts] Consider the system of the record and yellow jacket to determine the angular speed of the record when the yellow jacket has walked to the edge (i.e. a distance $R$ from the origin). You can assume that the torque on the record about a point $A$ located at the origin is zero and that the yellow jacket can be treated as a point mass.

D. [5 pts] When the yellow jacket reaches the edge of the record she is thrown from the record. A few moments later you observe her to be at position $\vec{r} = <2R, R, 0>$ relative to the origin. At this instant the yellow jacket is moving with velocity $\vec{v} = <a, 0, 0>$ where $a$ is a positive constant. Calculate the yellow jacket's total vector angular momentum relative to a point $B$ located at $\vec{r}_B = <R, 0, 0>$. The yellow jacket is so small it can be considered a point mass.
Problem 2

C. [3 pts] A ball is moving in the y direction with a momentum \( \mathbf{p} \) (as shown in the figure). \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{C} \) are possible origins with \( \mathbf{r}_A, \mathbf{r}_B \) and \( \mathbf{r}_C \) the vector positions from those origins to the ball (\( \mathbf{B} \) origin is directly below the ball). Where \( \phi \) and \( \theta \) are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use \( \mathbf{B} \) as your origin.

D. [3 pts] A ball is moving in the y direction with a momentum \( \mathbf{p} \) (as shown in the figure). \( \mathbf{A}, \mathbf{B} \) and \( \mathbf{C} \) are possible origins with \( \mathbf{r}_A, \mathbf{r}_B \) and \( \mathbf{r}_C \) the vector positions from those origins to the ball (\( \mathbf{B} \) origin is directly below the ball). Where \( \phi \) and \( \theta \) are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use \( \mathbf{C} \) as your origin.

E. [3 pts] Calculate the rotational angular momentum of the two balls of mass \( m \) each, connected by a rod of length \( d \), that rotates clockwise around the center of mass denoted by \( \mathbf{A} \) with an angular velocity \( \omega_1 \). You can assume the mass of the rod is zero.

F. [5 pts] Calculate the total angular momentum of the rotating barbell from part “E” if the center of mass is now connected to a bar of length \( b \), that rotates clockwise around an axes labeled \( \mathbf{B} \) with an angular velocity \( \omega_2 \). The barbell is still rotating clockwise about the barbells center of mass. You can assume the mass of the rods are zero.
Problem 1

A vinyl record has mass $M$ and radius $R$ and rotates about the center of the record with a constant angular velocity $\omega = -\omega_0 \hat{z}$ as indicated in the diagram. You notice a confused yellow jacket with mass $m$ is sitting on the record at the origin. At a time $t = 0$, the yellow jacket starts walking outward from the center of the record. From the perspective of the yellow jacket, she follows a straight line as indicated by the dashed yellow line in the diagram.

A. [5 pts] Calculate the total vector angular momentum of the record about a point $A$ located at the origin at time $t = 0$.

$\vec{L} = I \vec{\omega} = \left( \frac{1}{2} MR^2 \right) ( - \omega_0 \hat{z} )$

+3 for angular momentum equation
+2 for correct moment of inertia

$\vec{L} = - \frac{1}{2} MR^2 \omega_0 \hat{z}$

B. [5 pts] Calculate the total vector angular momentum of the yellow jacket about a point $A$ located at the origin at time $t = 0$ (i.e. when the yellow jacket is at the center of the record). The yellow jacket is so small it can be considered a point mass.

None because it is located at the origin

$\vec{L} = \langle 0, 0, 0 \rangle$
C. [10 pts] Consider the system of the record and yellow jacket to determine the angular speed of the record when the yellow jacket has walked to the edge (i.e. a distance $R$ from the origin). You can assume that the torque on the record about a point $A$ located at the origin is zero and that the yellow jacket can be treated as a point mass.

Conservation of angular momentum

$$\Delta \vec{L} = 0$$

$$\vec{L}_f - \vec{L}_i = 0$$

$$I_f \vec{\omega}_f + \frac{1}{2} M R^2 \omega_o \hat{z} = 0$$

$$|\vec{\omega}_f| = \frac{\frac{1}{2} MR^2 \omega_o}{I_f}$$

$$I_f = \frac{1}{2} MR^2 + m R^2$$

$$= (\frac{1}{2} M + m) R^2$$

$$|\vec{\omega}_f| = \frac{\frac{1}{2} M R^2 \omega_o}{(\frac{1}{2} M + m) R^2}$$

$$|\vec{\omega}_f| = \frac{M}{(M + m)} \omega_o$$

-1 Clerical / Units
-2 Math Error / Minor Physics Error
-4 Major Physics Error
-8 BTN

Common mistakes:
-2 Incorrect moment of inertia

""Watch for POE"

D. [5 pts] When the yellow jacket reaches the edge of the record she is thrown from the record. A few moments later you observe her to be at position $\vec{r} = <2R, R, 0>$ relative to the origin. At this instant the yellow jacket is moving with velocity $\vec{v} = <a, 0, 0>$ where $a$ is a positive constant. Calculate the yellow jacket’s total vector angular momentum relative to a point $B$ located at $\vec{r}_B = <R, 0, 0>$. The yellow jacket is so small it can be considered a point mass.

$$\vec{L} = \vec{r} \times \vec{p}$$

$$\vec{r}' = \vec{r} - \vec{r}_B = <R, R, 0>$$

$$\vec{p} = m \vec{v} = <ma, 0, 0>$$

$$\vec{L} = -ma \hat{R} \hat{z}$$

+1 for having $L=\vec{r} \times \vec{p}$
+2 for correct $r$
+2 for correctly calculating cross product
C. [3 pts] A ball is moving in the y direction with a momentum $\vec{p}$ (as shown in the figure). A, B and C are possible origins with $\vec{r}_A$, $\vec{r}_B$ and $\vec{r}_C$ the vector positions from those origins to the ball (B origin is directly below the ball). Where $\phi$ and $\theta$ are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use B as your origin.

**All or nothing**

D. [3 pts] A ball is moving in the y direction with a momentum $\vec{p}$ (as shown in the figure). A, B and C are possible origins with $\vec{r}_A$, $\vec{r}_B$ and $\vec{r}_C$ the vector positions from those origins to the ball (B origin is directly below the ball). Where $\phi$ and $\theta$ are the angles between the position vectors and the vertical. Calculate the magnitude of the translational angular momentum if you use C as your origin.

$$\vec{L} = |\vec{r} \times \vec{p}| = r_c |\vec{p}| \sin \theta$$

$$\vec{L} = r_c |\vec{p}| \sin \theta$$

**All or nothing**
E. [3 pts] Calculate the rotational angular momentum of the two balls of mass $m$ each, connected by a rod of length $d$, that rotates clockwise around the center of mass denoted by $A$ with an angular velocity $\omega_1$. You can assume the mass of the rod is zero.

\[
\vec{L} = I \vec{\omega} = -2m \left( \frac{d}{2} \right)^2 \omega_1 \hat{z}
\]

\[
\vec{L} = -\frac{md^3}{2} \omega_1 \hat{z}
\]

**All or nothing**

F. [5 pts] Calculate the total angular momentum of the rotating barbell from part “E” if the center of mass is now connected to a bar of length $b$, that rotates clockwise around an axes labeled $B$ with an angular velocity $\omega_2$. The barbell is still rotating clockwise about the barbells center of mass. You can assume the mass of the rods are zero.

\[
\vec{L} = \vec{L}_{\text{rot}} + \vec{L}_{\text{trans}}
\]

\[
\vec{L} = -\frac{md^3}{2} \omega_1 \hat{z} - 2mb^2 \omega_2 \hat{z}
\]

\[
\vec{L} = -\left( \frac{md^3}{2} \omega_1 + 2mb^2 \omega_2 \right) \hat{z}
\]

+2 for recognizing sum of angular momentums
+3 for correct moment of inertia